TNS and non-equilibrium dynamics

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Miguel Frías-Pérez, Luca Tagliacozzo (IFF)

PRB 106, 115117 (2022)
arXiv:2308.04291

MAX PLANCK INSTITUTE
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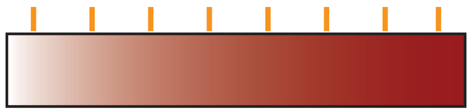
TNRG 31.10.2023

TNS in quantum many-body systems

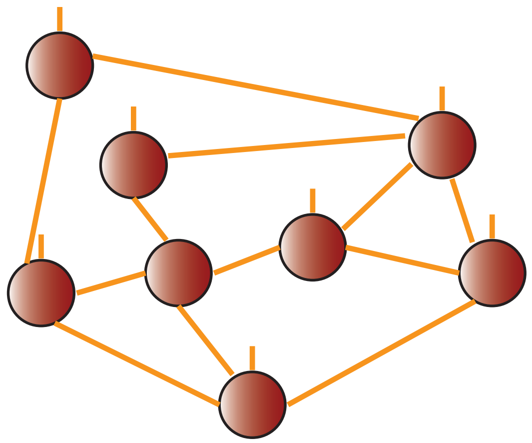
A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} \boxed{c_{i_1 \ldots i_N}} |i_1 \ldots i_N\rangle$$



N-legged tensor

d^N



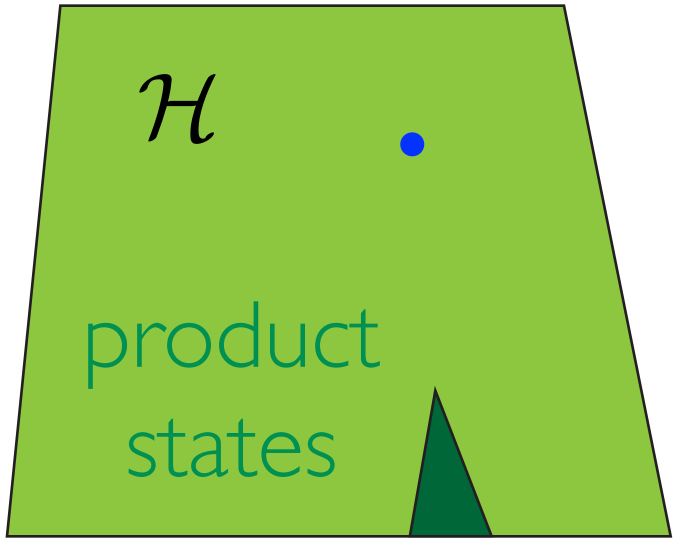
$\mathrm{poly}(N)$

A TNS has only a polynomial number of parameters

WHY SHOULD TNS BE USEFUL?

States appearing in Nature are peculiar

State at random from Hilbert space is not close to product



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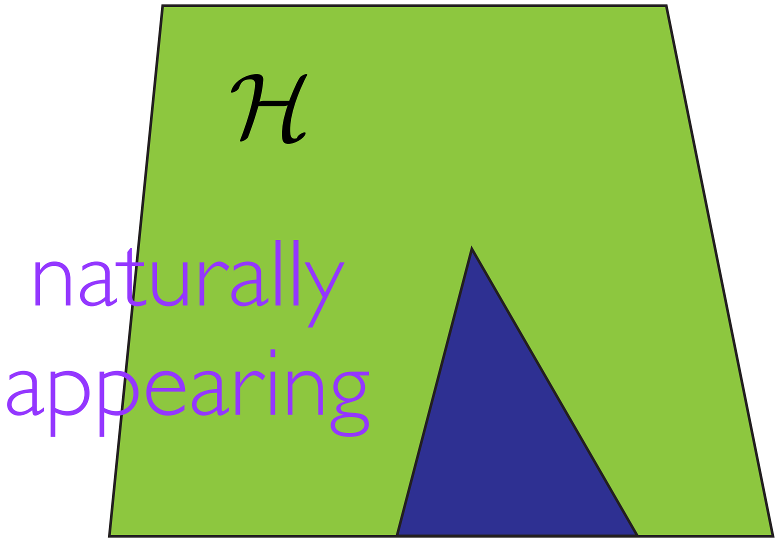
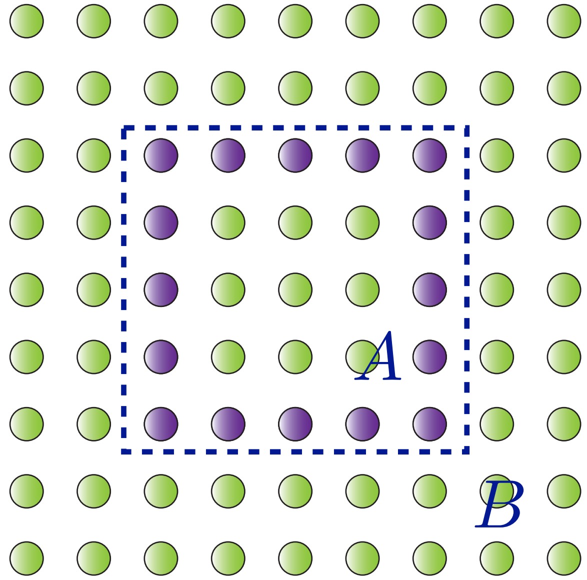


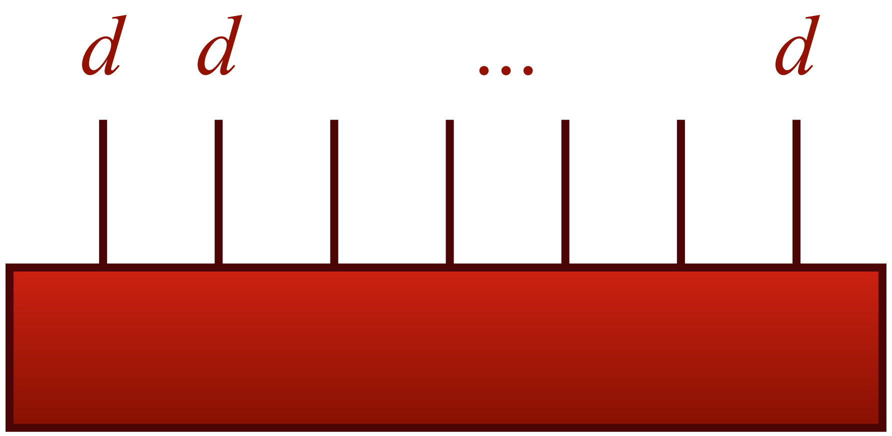
area law



We look for states with *little* entanglement

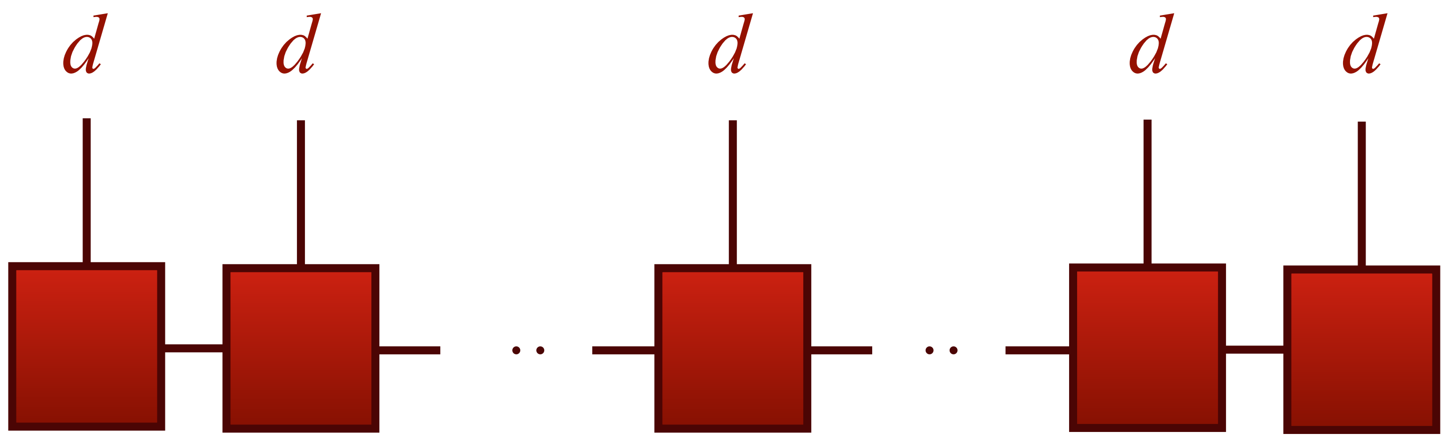
TNS = entanglement based ansatz

MPS

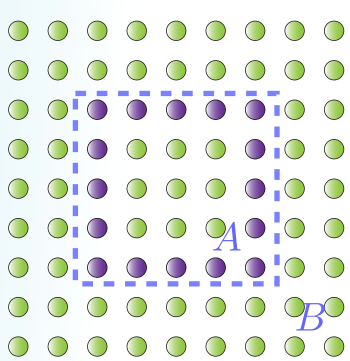


$$|\Phi\rangle = \sum_{s_1,\ldots,s_N=1}^{d} c_{s_1,\ldots,s_N} |s_1,\ldots,s_N\rangle$$

MPS



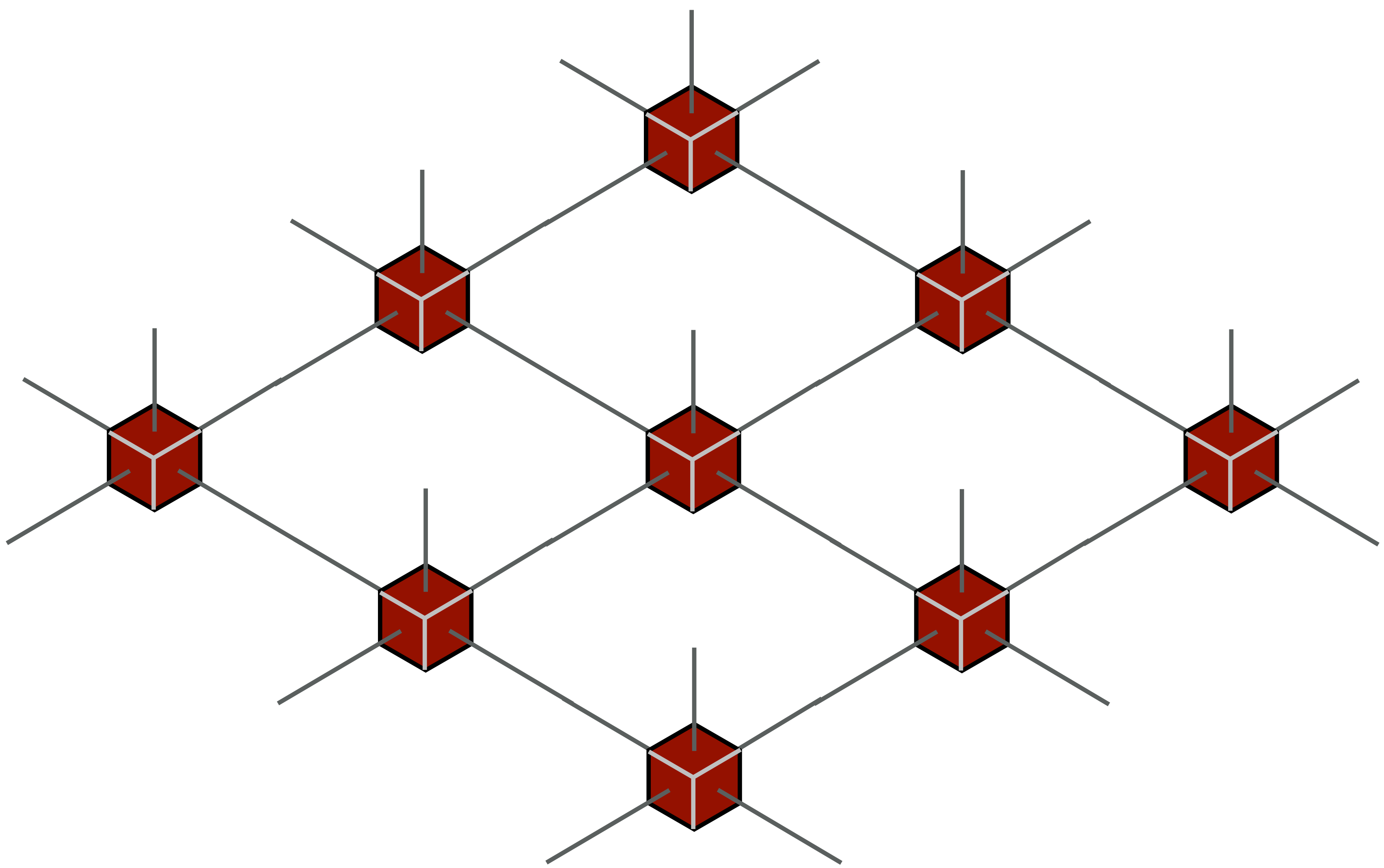
$$|\Phi_D\rangle = \sum_{s_1,\ldots,s_N=1}^{d} \mathrm{tr}\left(A^{s_1}[1]\ldots A^{s_1}[N]\right)|s_1,\ldots,s_N\rangle$$

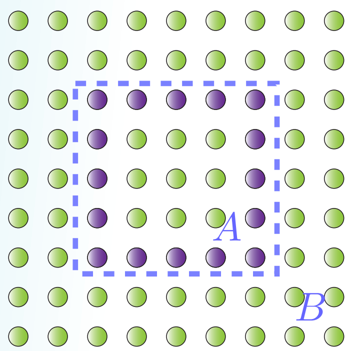


area law by construction

$$S(L/2) \leq \log D$$

PEPS



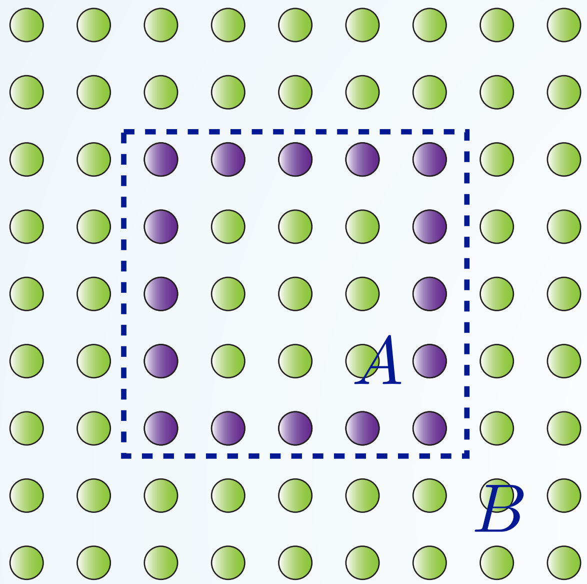


area law by construction

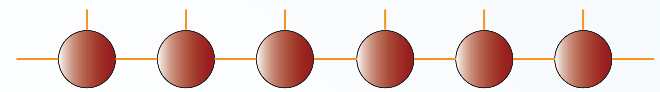
Verstraete, Cirac, 2004

TNS = entanglement based ansatz

area law

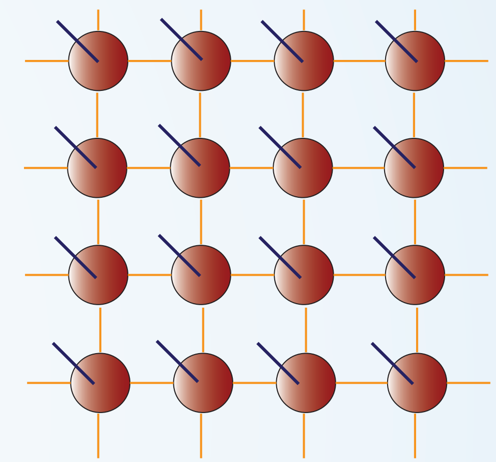


MPS



Schollwöck Ann.Phys.2011

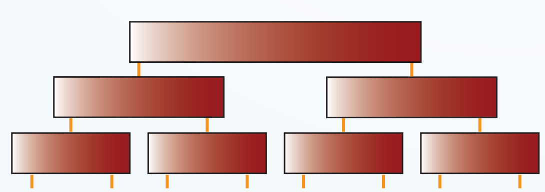
PEPS



Verstraete et al. Adv. Phys. 2008

other TNS

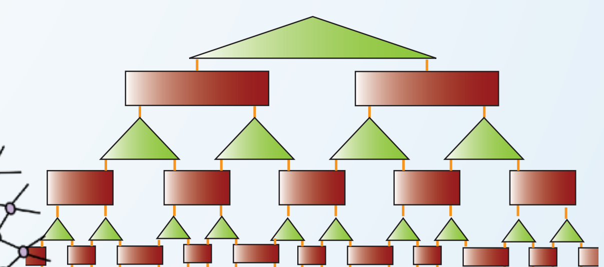
TTN

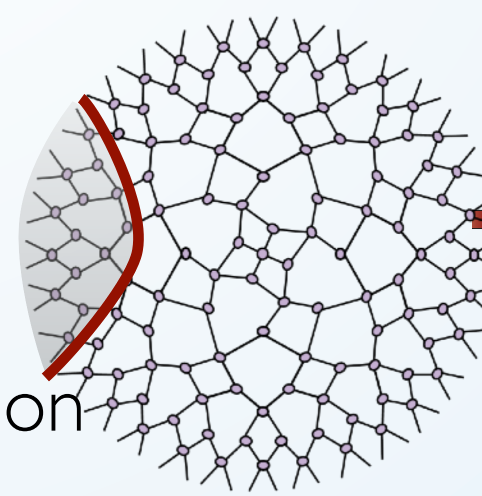


Shi et al PRA 2006

Vidal PRL 2007

MERA





suggested connection
to AdS/CFT

Swingle PRD 2012
Molina JHEP 2013
Nozaki et al JHEP 2012
Bao et al PRD 2015

TNS are very useful in the quantum many-body context…

works for GS, low energy, thermal equilibrium…

Verstraete, Cirac, PRB 2006 Hastings PRB 2006
Hastings J. Stat. Phys 2007 Molnar *et al.* PRB 2015

area laws

but not for high energy eigenstates, quenches…

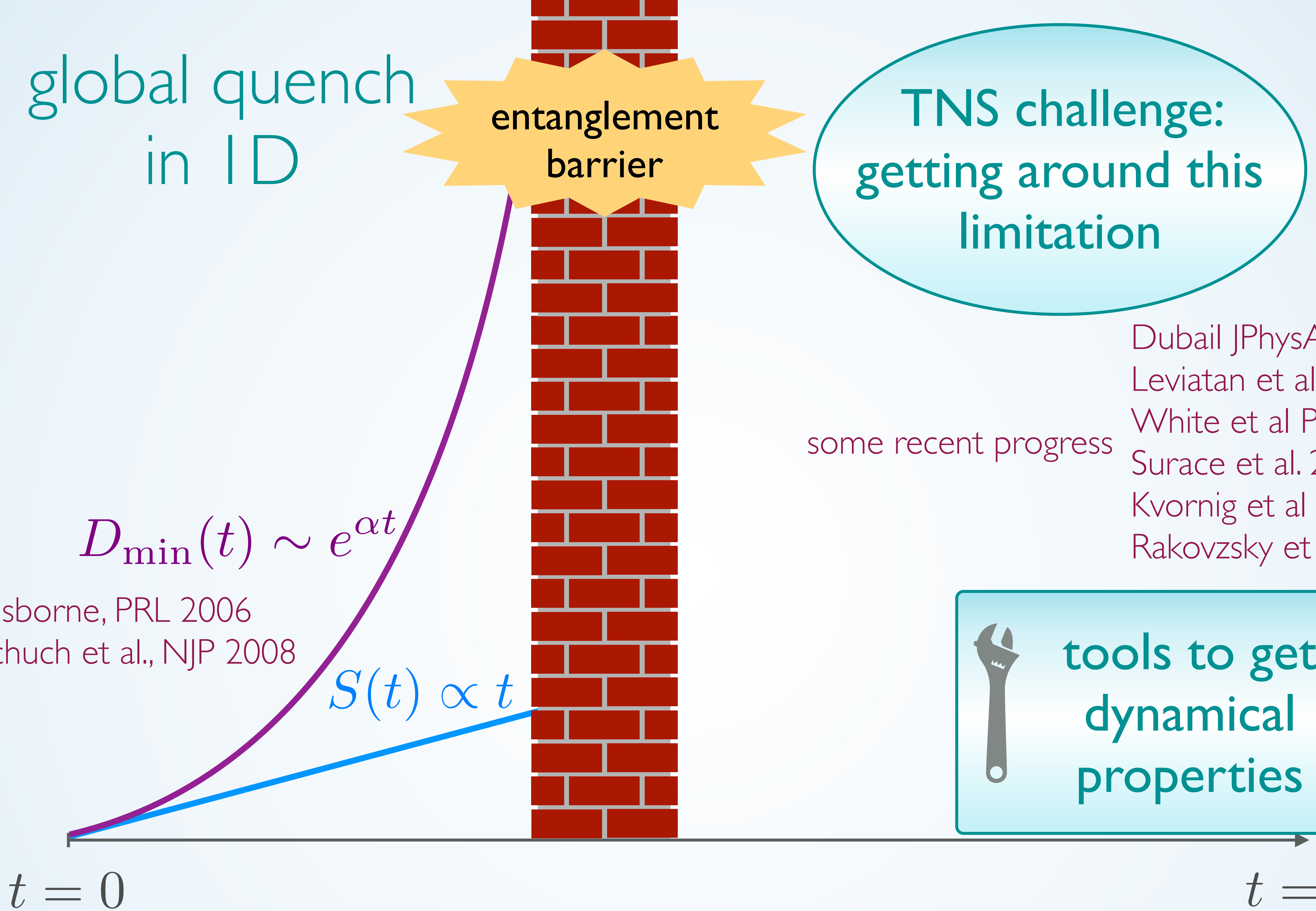
Osborne, PRL 2006 Vidmar *et al.*, PRL 2017
Schuch *et al.*, NJP 2008

volume law

entanglement growth in non-equilibrium scenarios limits the applicability of MPS

fundamental questions: thermalization, ETH…

global quench in 1D



Osborne, PRL 2006
Schuch et al., NJP 2008

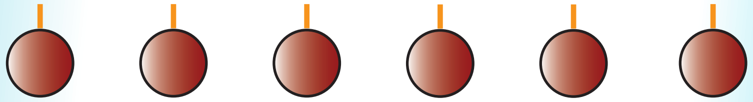
TNS challenge: getting around this limitation

some recent progress

Dubail JPhysA 2017
Leviatan et al. 2017
White et al PRB 2018
Surace et al. 2018
Kvornig et al 2021
Rakovzsky et al 2022

tools to get dynamical properties

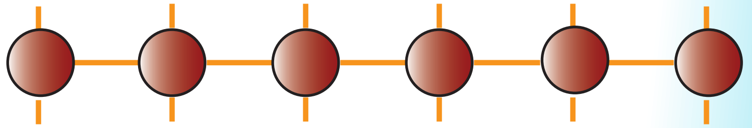
product state



easy to write as MPS

local observables

thermal states



well approximated as MPO

alternative: give up description of the full state

1 light-cone TN for non-equilibrium evolution of local observables

M. Frías-Pérez, MCB, PRB 106, 115117 (2022)

2 transforming long-range entanglement into mixture

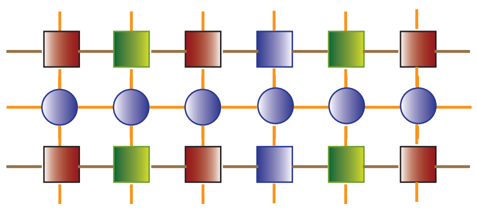
M. Frías-Pérez, L. Tagliacozzo, MCB, arXiv:2308.04291

3 spectral properties of the QMB Hamiltonian

Yang et al. PRL124, 100602 (2020),
Lu, MCB, Cirac, PRX Quantum 2, 02032 (2021)
Yang, Cirac, MCB, PRB 106, 024307 (2022)

give up description of the full state: (local) operators

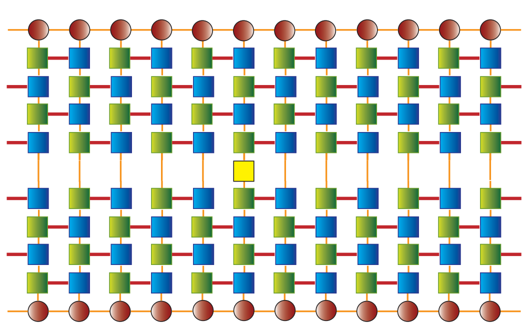
evolving operators: Heisenberg picture Hartmann et al, PRL 2009



also for mixed states
operator space entanglement

Prosen Pizorn, PRL 2008

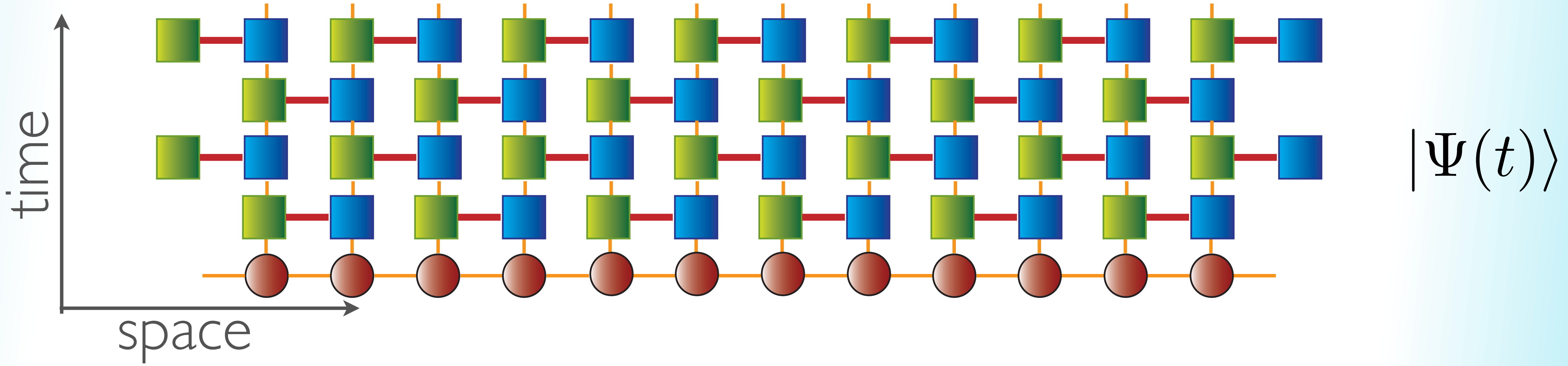
observables as TN to contract



different *entanglement* quantities

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes et al., NJP 2012
Hastings, Mahajan 2014
Frías-Pérez, MCB PRB 2022

time-dependent observable as a TN

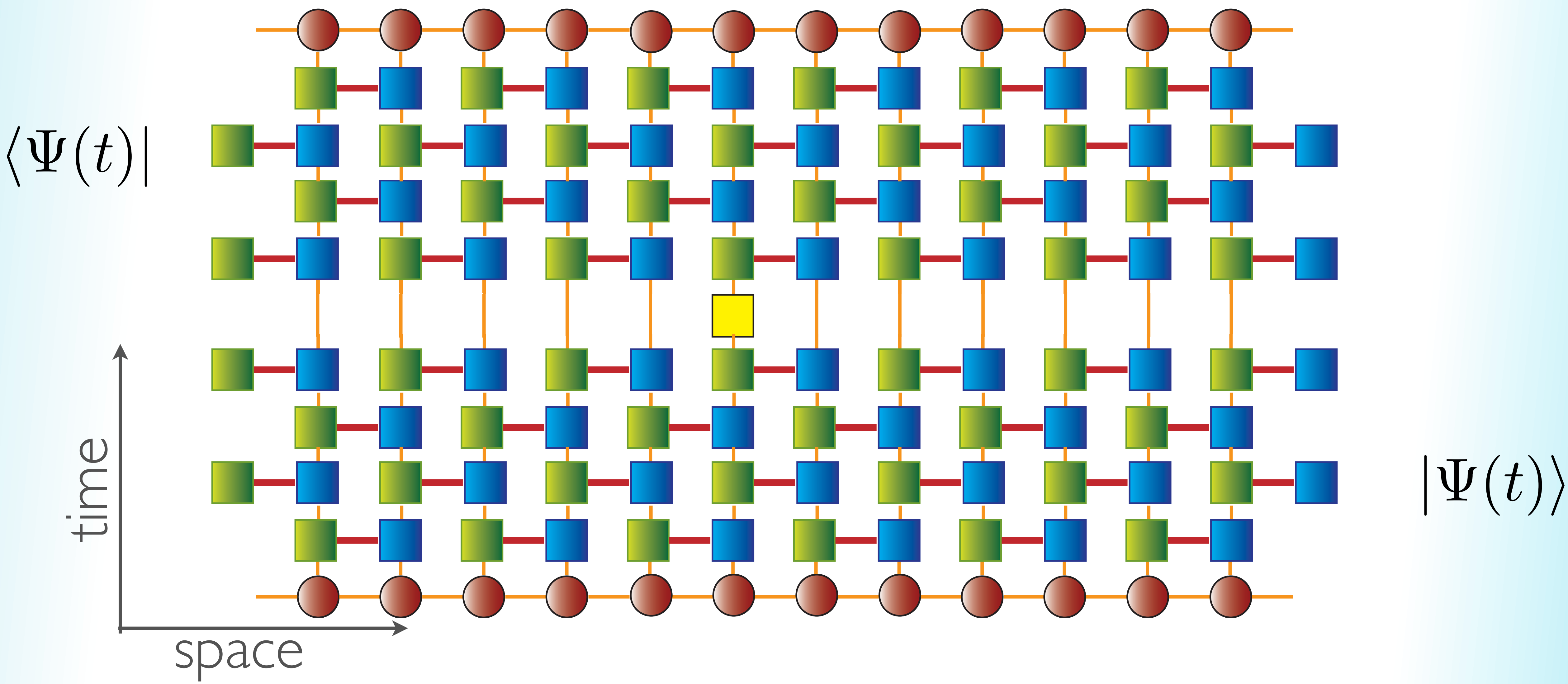


time-dependent observable as a TN

TN describe observables, not states

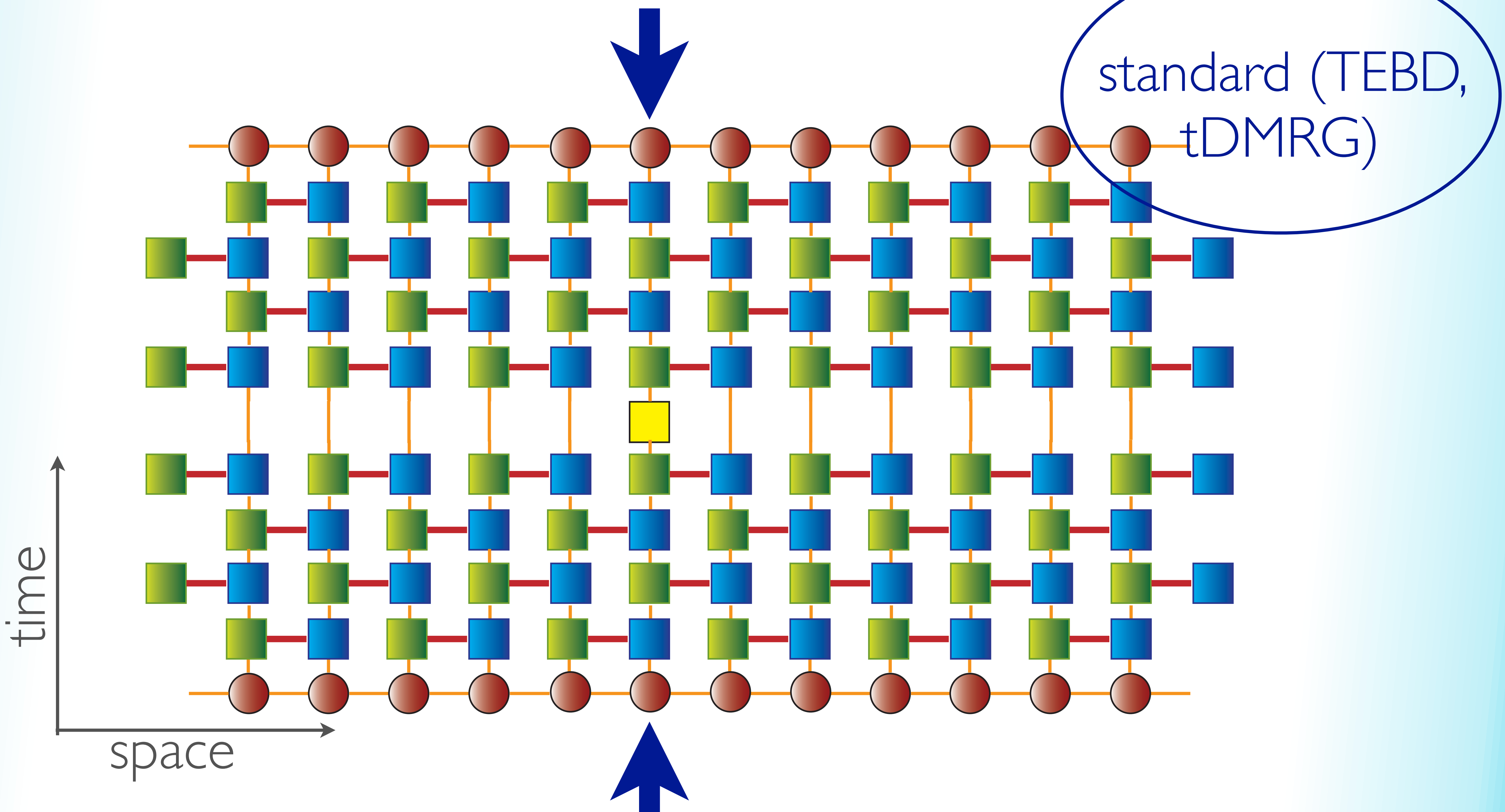
exact contraction not possible
#P complete

$$\langle\Psi(t)|O|\Psi(t)\rangle$$



time-dependent observable as a TN

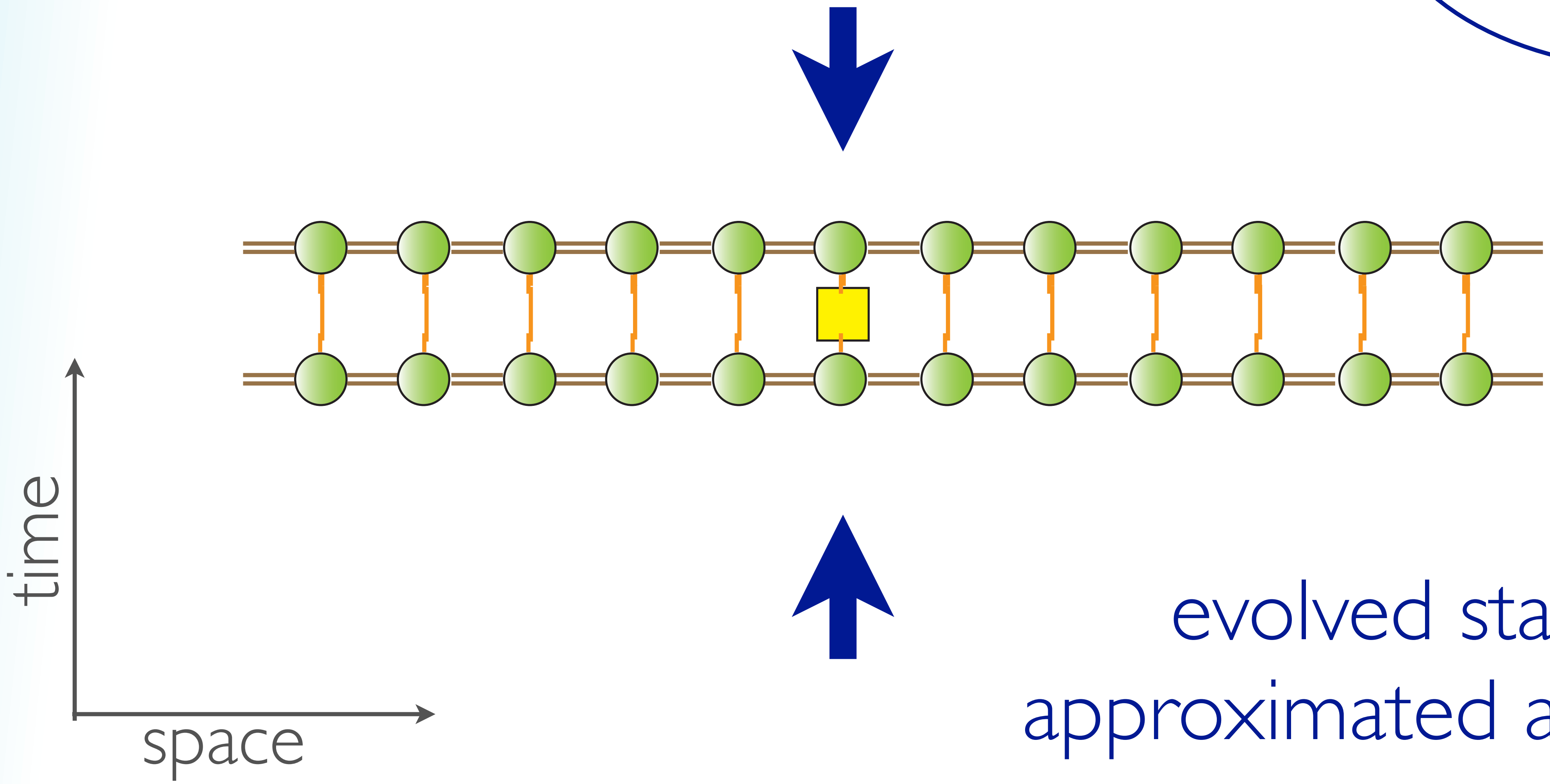
different approximate contraction strategies



time-dependent observable as a TN

different approximate contraction strategies

standard (TEBD, tDMRG)



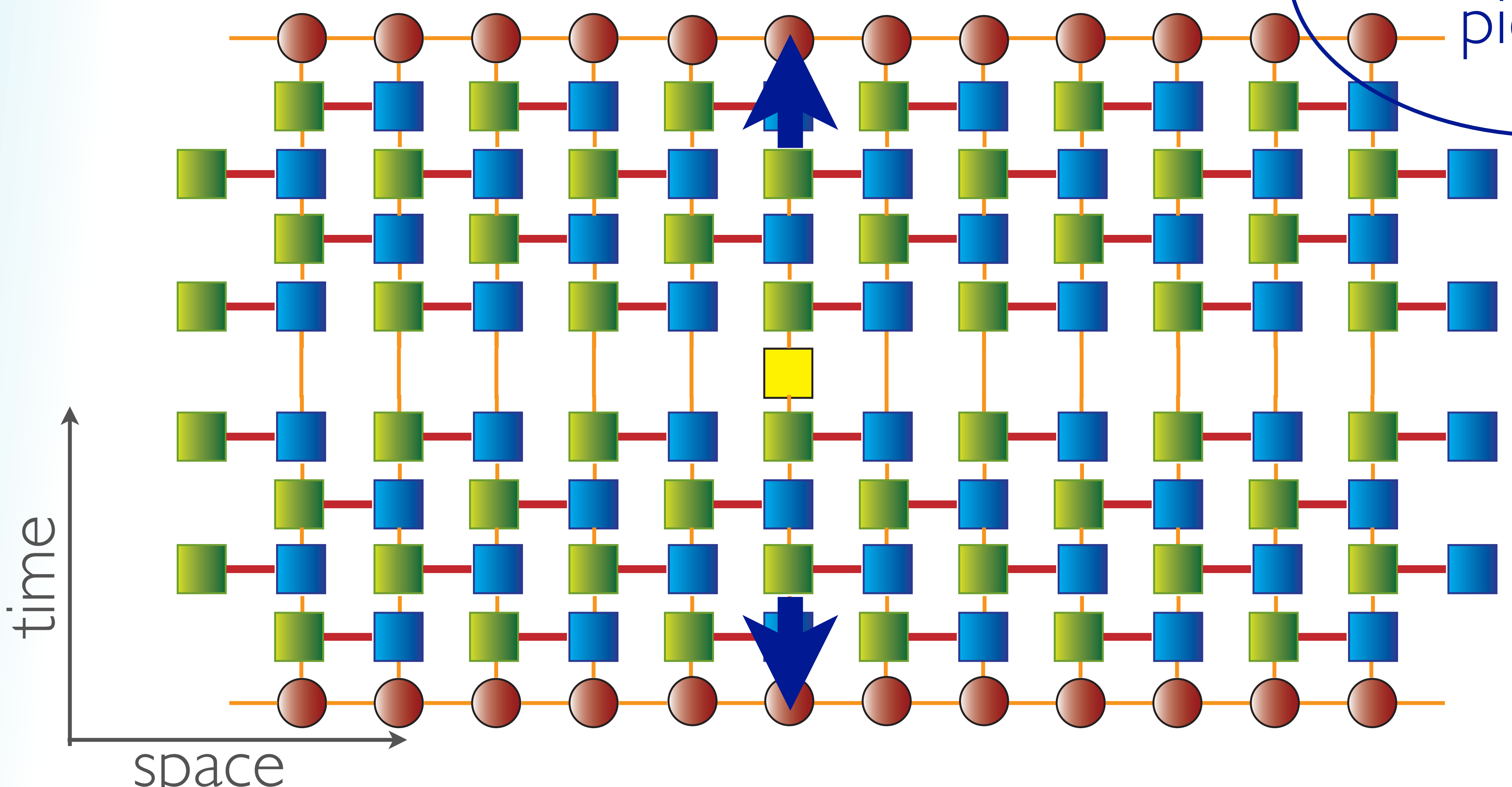
evolved state approximated as MPS

time-dependent observable as a TN

different approximate contraction strategies

evolved operator as MPO

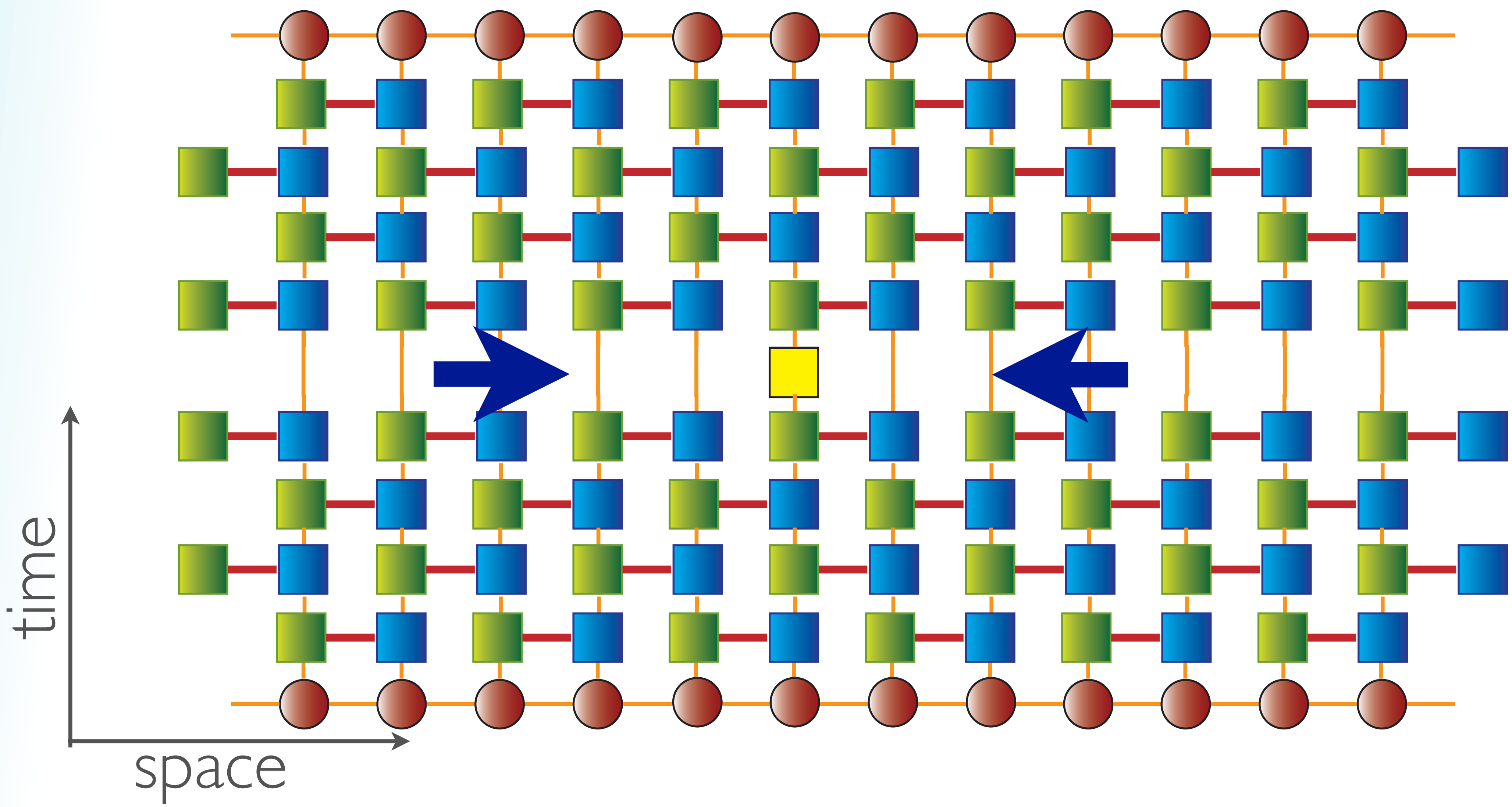
Heisenberg picture



time-dependent observable as a TN

for infinite systems, transverse folding approach

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes, Cirac, MCB, NJP 2012
Hastings, Mahajan 2014



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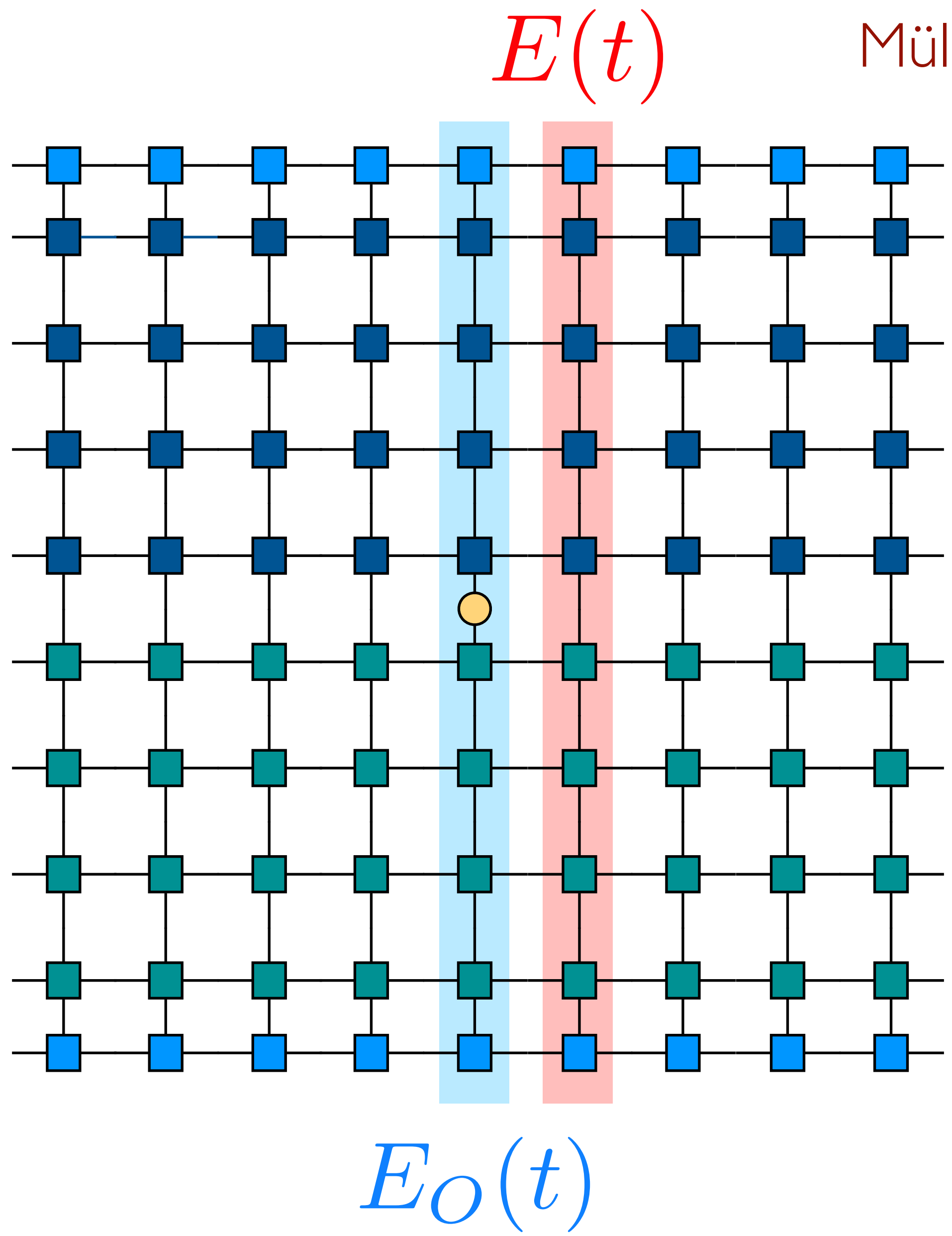

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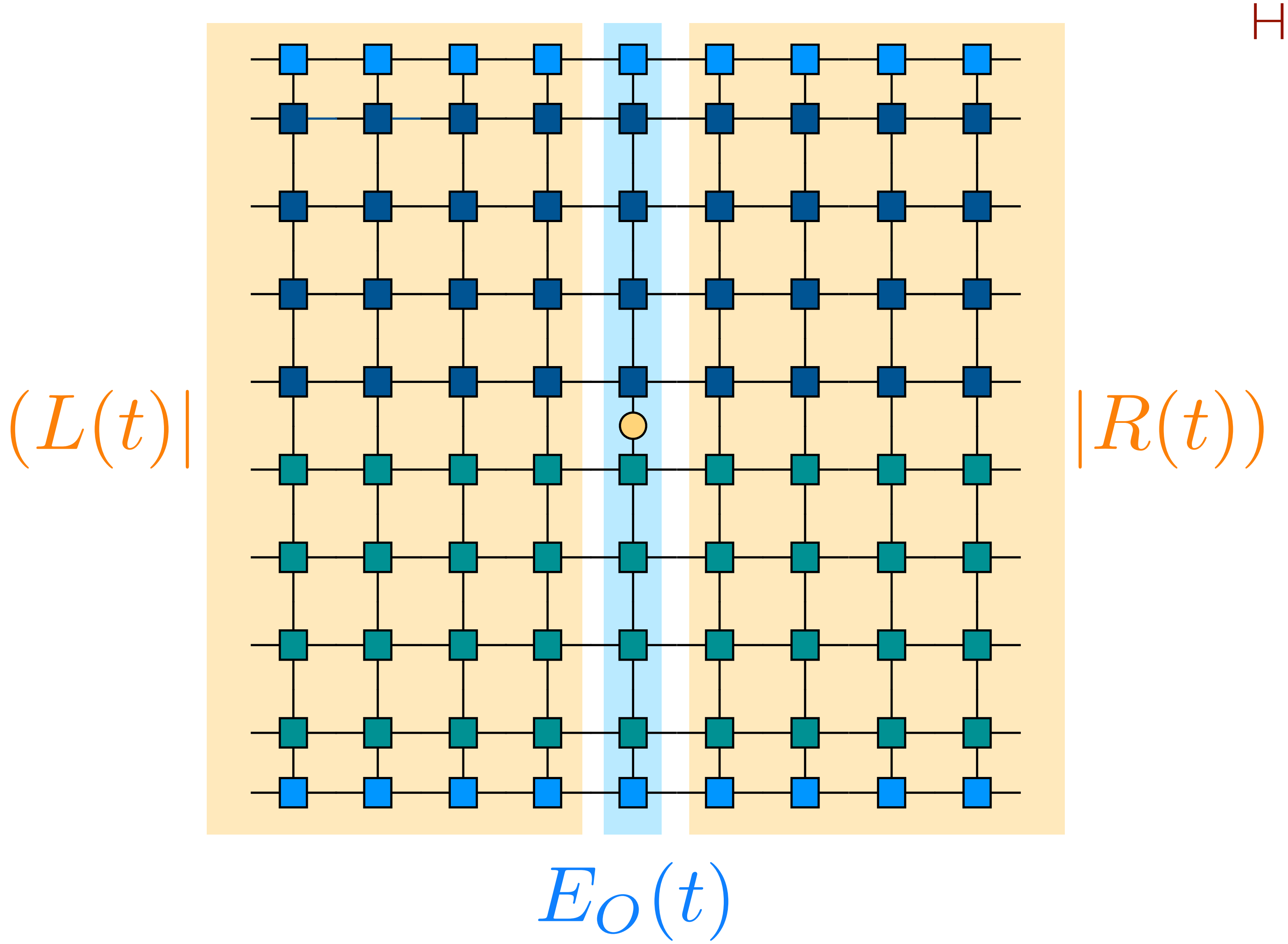

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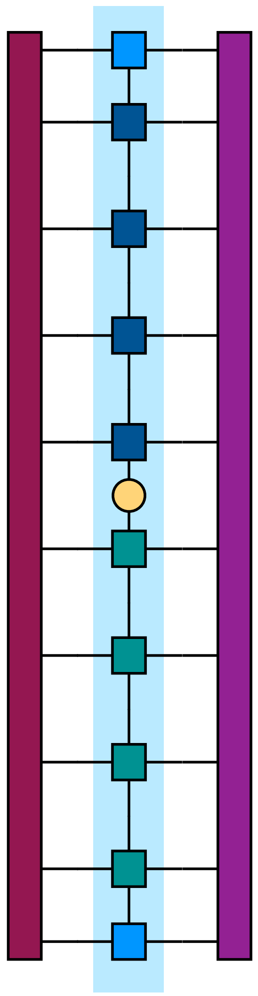
$(L(t)|$



$|R(t))$

$E_O(t)$

transverse folding approach

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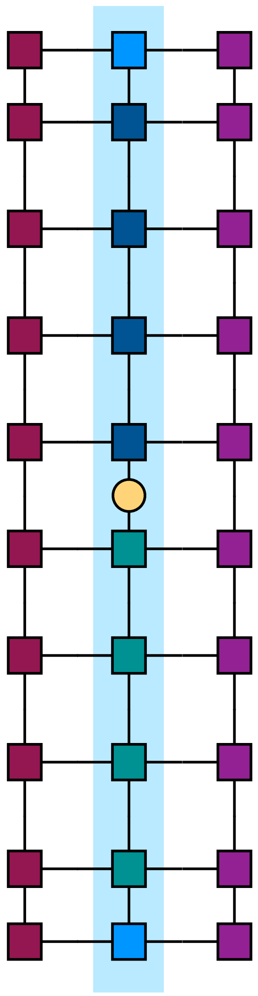

$|R(t))$

$E_O(t)$

transverse folding approach

intuition: model free propagating excitations

MCB, Hastings, Verstraete, Cirac, PRL 2009
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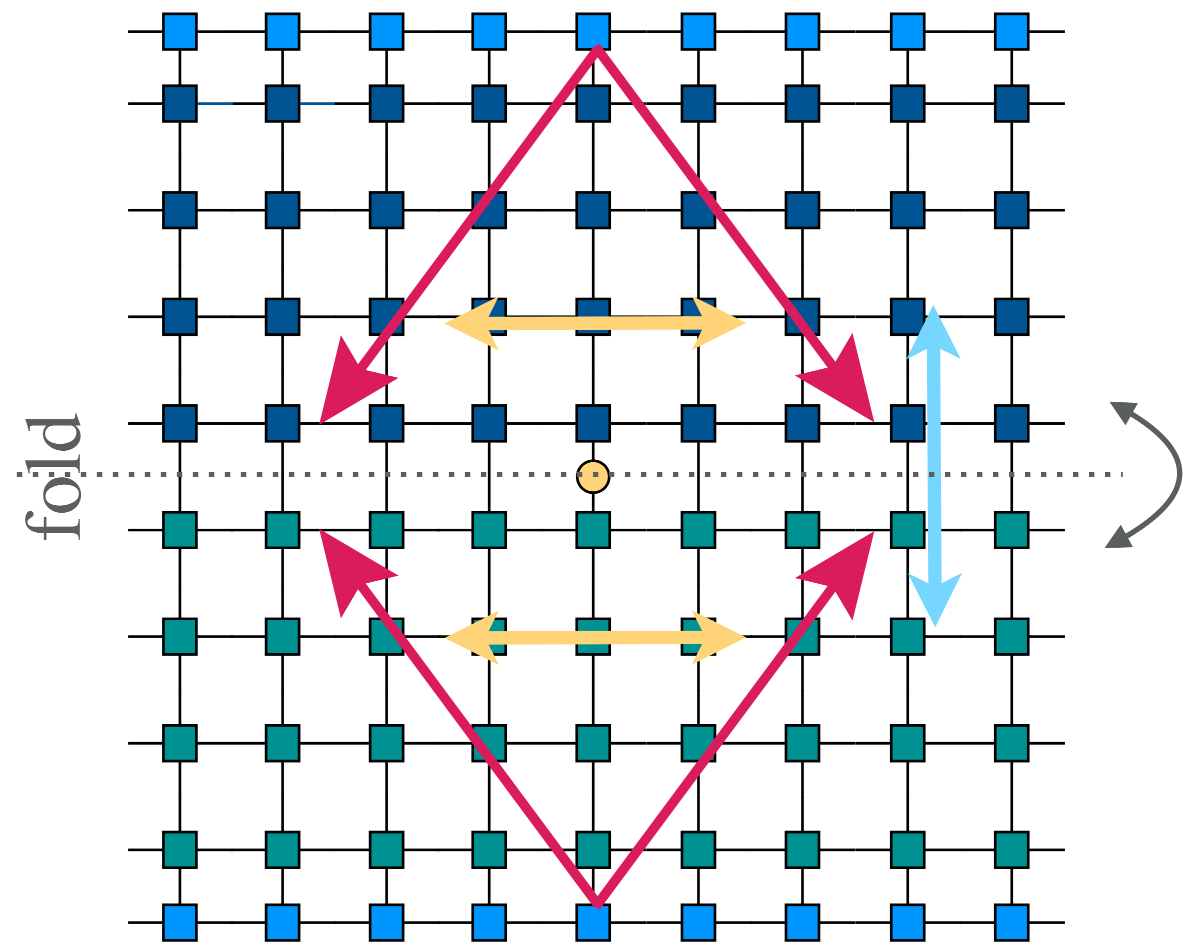
Hastings, Mahajan 2014

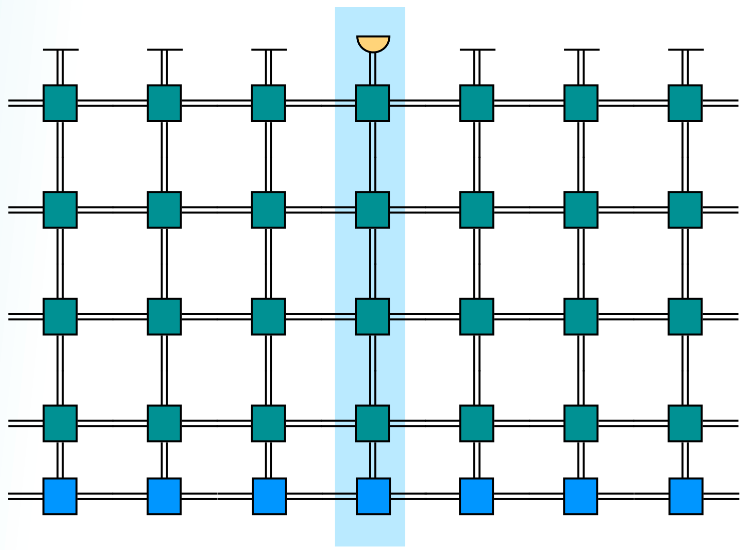


transverse folding approach

free propagating excitations

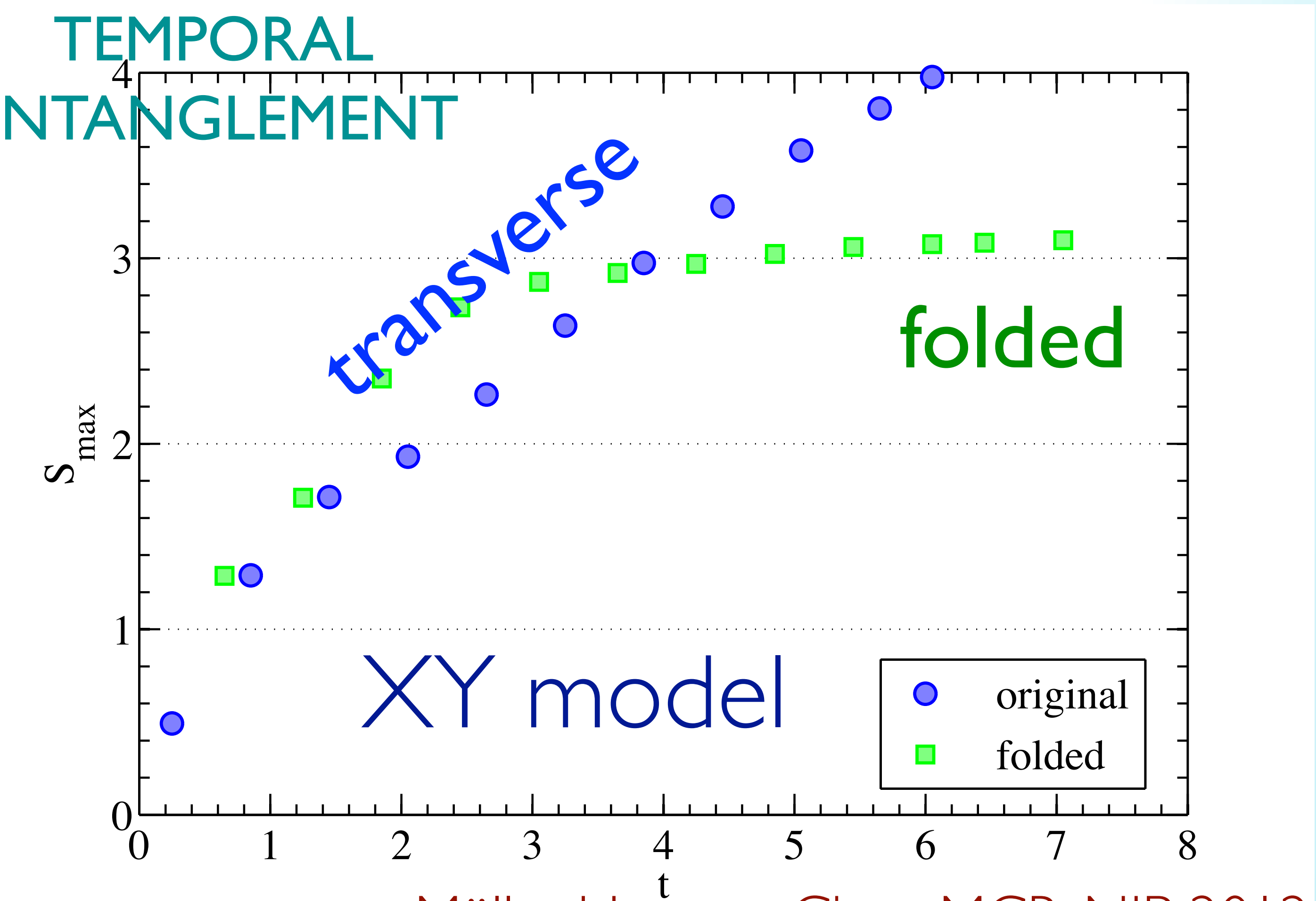
recently: influence functional
Sonner et al, Ann. Phys 2021
Lerose et al. PRX 2021
Ye, Chan, J. Chem. Phys. 2021

see also Carignano 2307.11649



$E_O(t)$

closest real case: global quench
in free fermionic models



Müller-Hermes, Cirac, MCB, NJP 2012
see also Giudice et al., PRL128, 220401 (2022)

transverse folding + light cone =TLCC

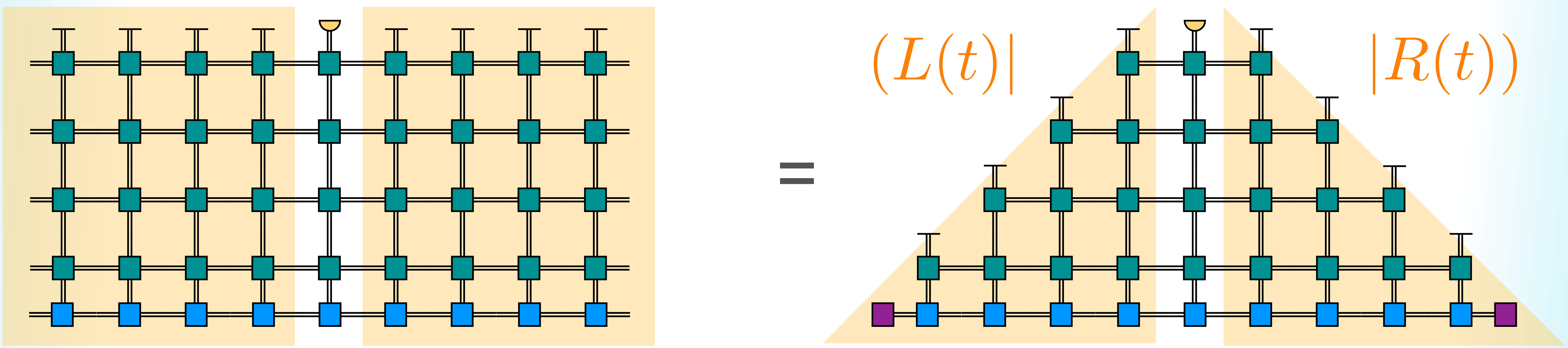
cancelling local unitaries

gain in efficiency

systematic increment of t

improved convergence with Hastings' truncation

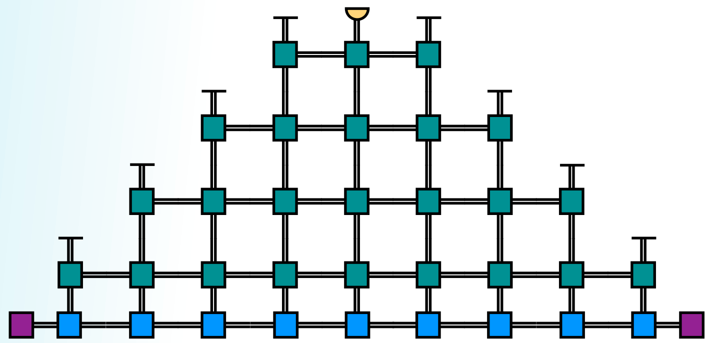
Hastings, Mahajan 2014



see also Lerose et al., PRB 107, L060305 (2023)

Frías-Pérez, MCB, PRB 106, 115117 (2022)

transverse folding + light cone =TLCC

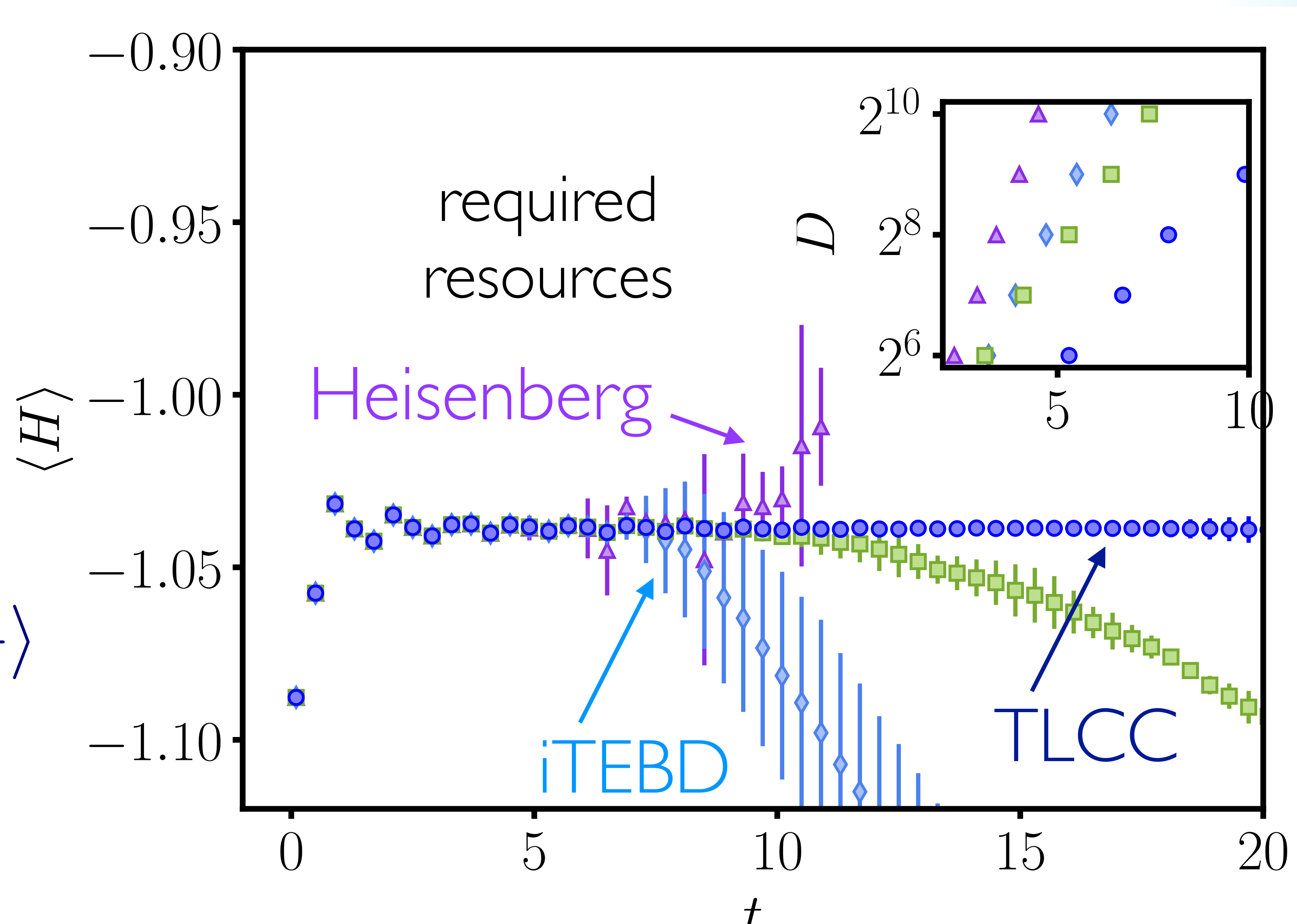


$$H_{\text{Ising}} = J \sum_{i=1}^{N-1} \sigma_z^{[i]} \sigma_z^{[i+1]} + g \sum_{i}^{N} \sigma_x^{[i]} + h \sum_{i}^{N} \sigma_z^{[i]}$$

$(J, g, h) =$

$(1, -1.05, 0.5)$

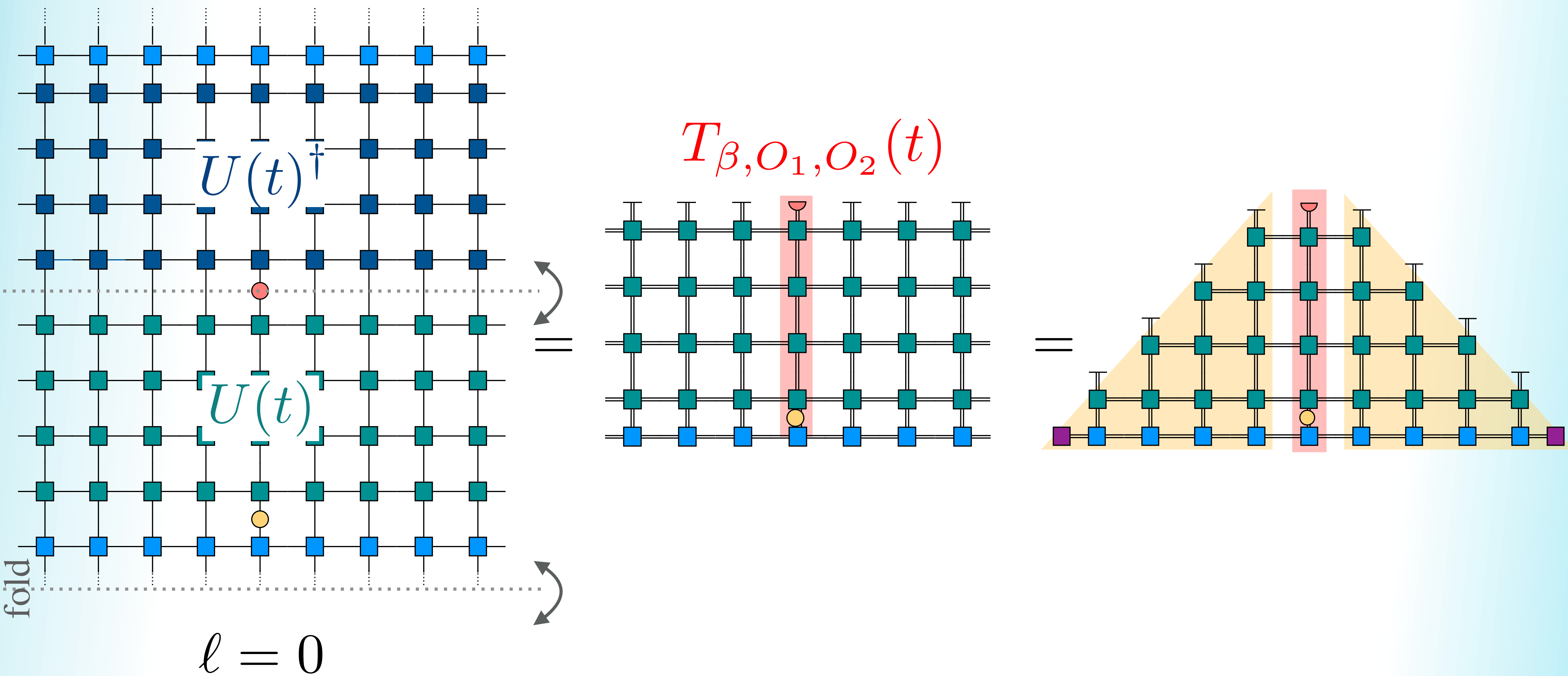
quench from $|X+\rangle$

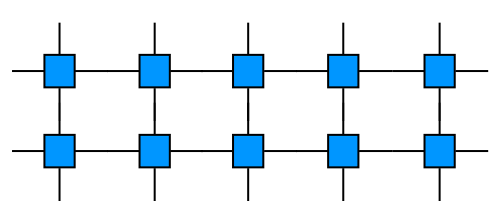


Frías-Pérez, MCB, PRB 106, 115117 (2022)

computing response functions

$$C_{1,2}(t,\ell,\beta) = \mathrm{tr}(\rho_\beta O_2^{[\ell]}(t) O_1^{[0]}(0))$$



$e^{-\beta H} \approx$ 

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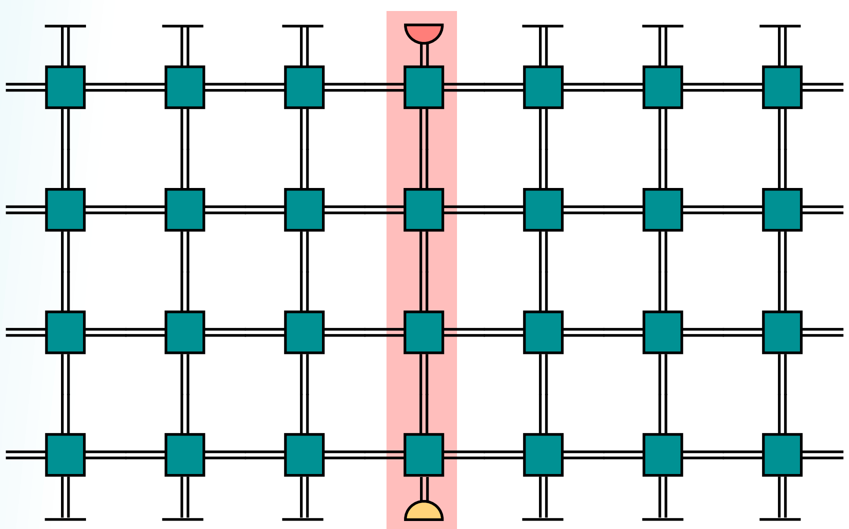
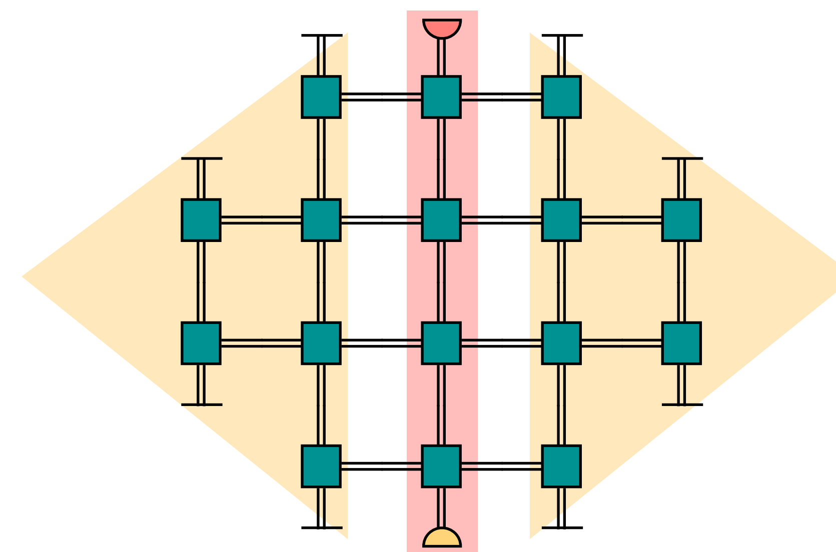
$\ell = 0$



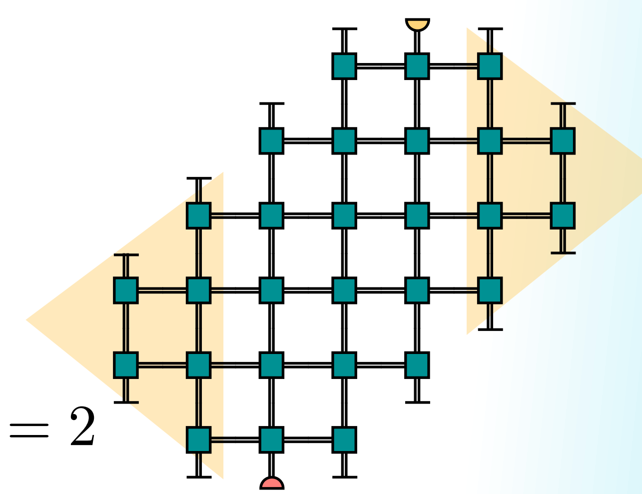
$=$



$T_{\beta=0,O_1,O_2}(t)$

infinite temperature

$\beta = 0$



$\ell = 2$

Frías-Pérez, MCB, PRB 106, 115117 (2022)

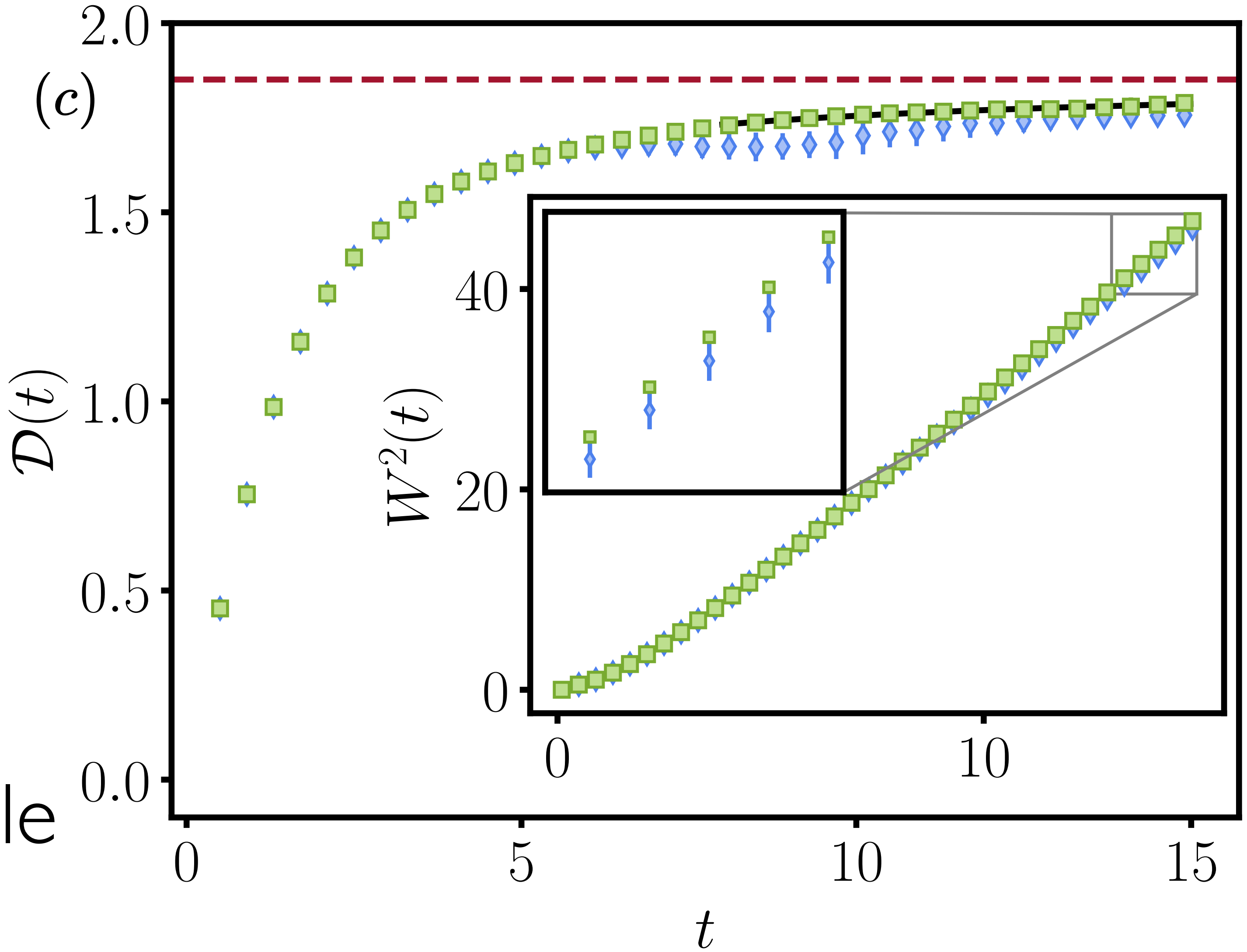
computing response functions

tilted Ising

energy density

infinite temperature

$$O_E^{[i]} := J\sigma_i^z\sigma_{i+1}^z + \frac{g}{2}(\sigma_i^x + \sigma_{i+1}^x) + \frac{h}{2}(\sigma_i^z + \sigma_{i+1}^z)$$



diffusion constant

non-integrable
$(1, -1.05, 0.5)$

Frías-Pérez, MCB, PRB 106, 115117 (2022)

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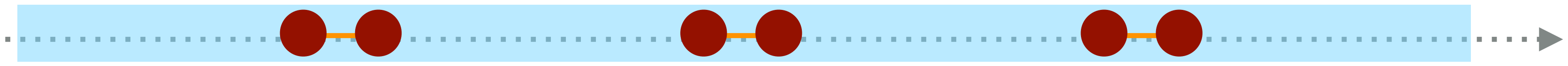
Yang, Cirac, MCB, PRB 106, 024307 (2022)

back to quasiparticle intuition



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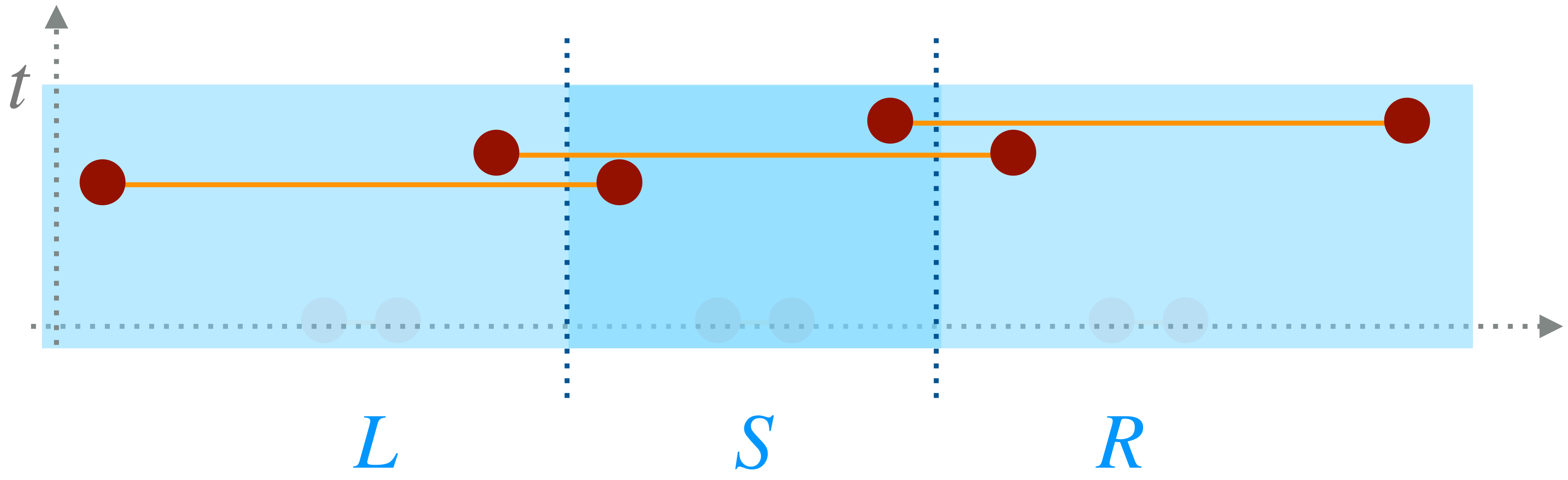

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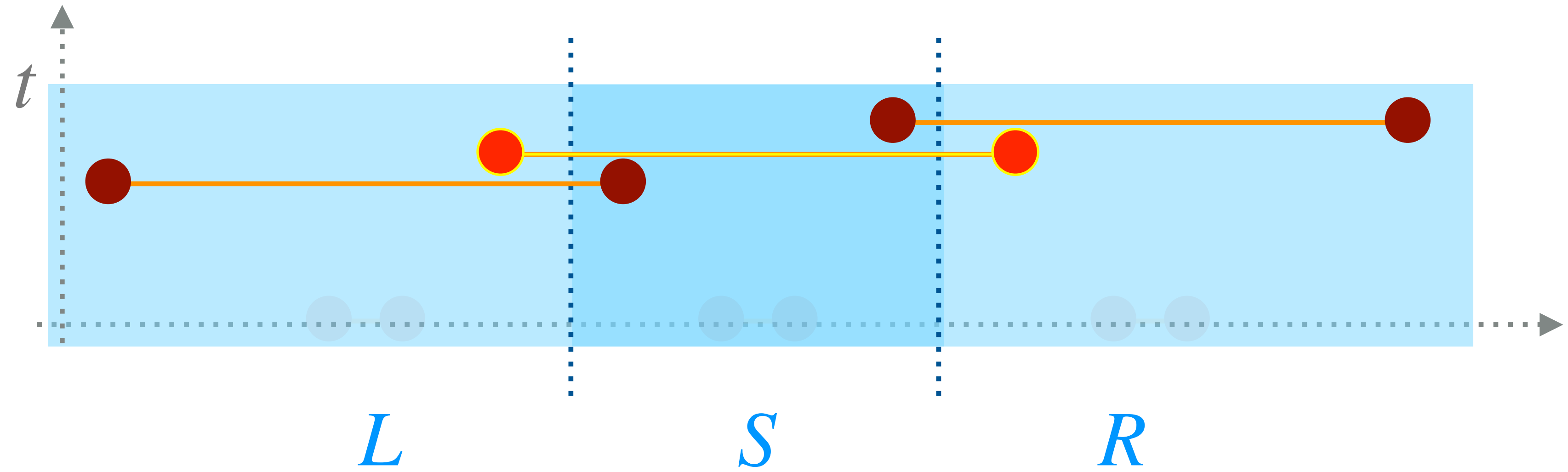
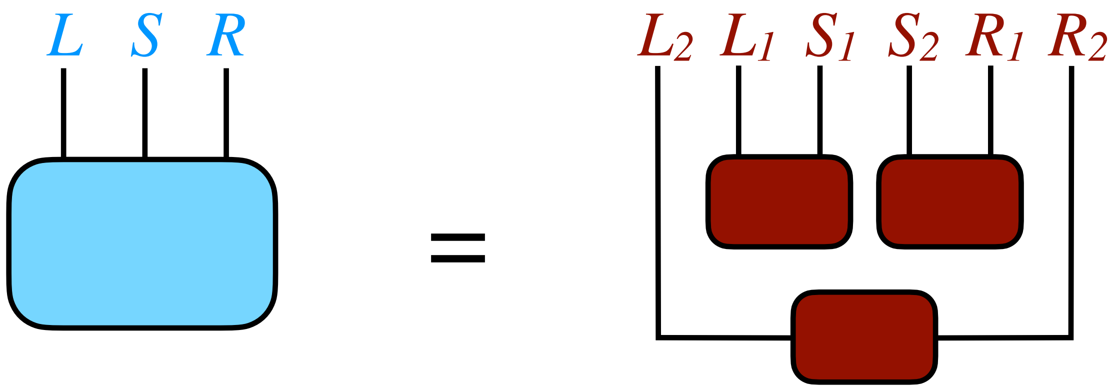


$$|\Psi_{LSR}\rangle = |\phi_{L_1S_1}\rangle \otimes |\phi_{S_2R_1}\rangle \otimes |\phi_{L_2R_2}\rangle$$



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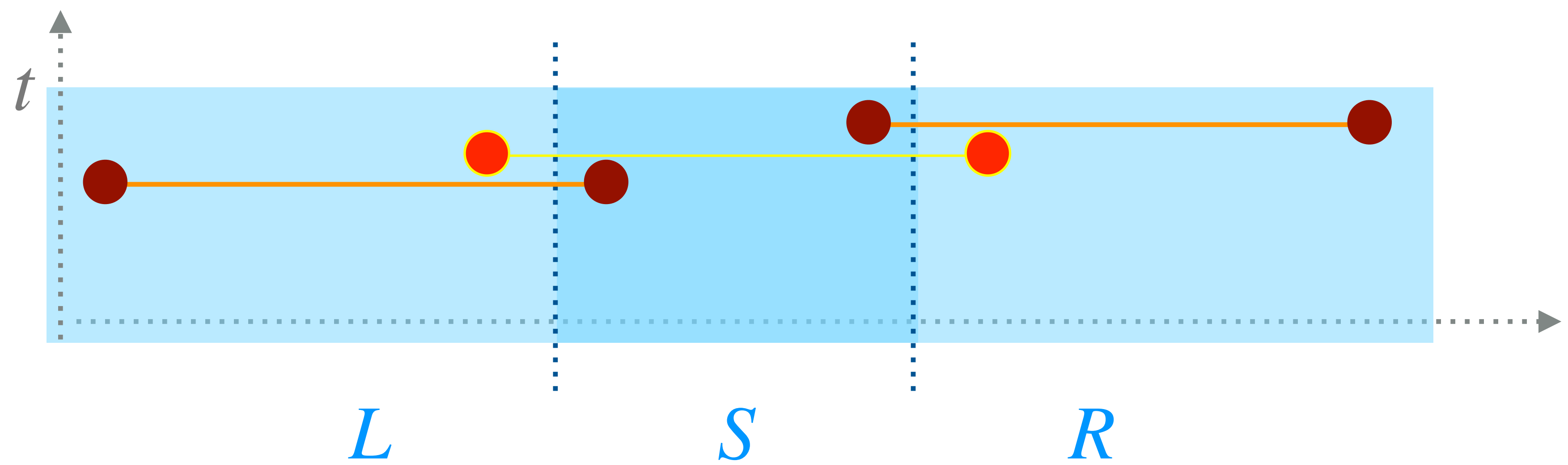
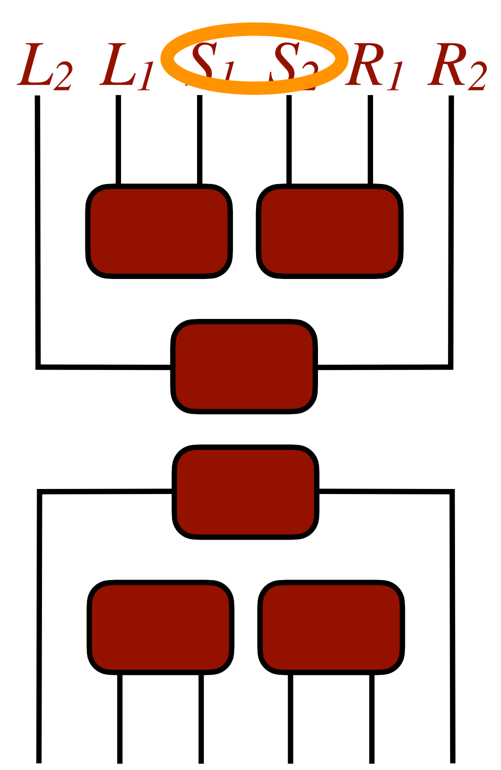
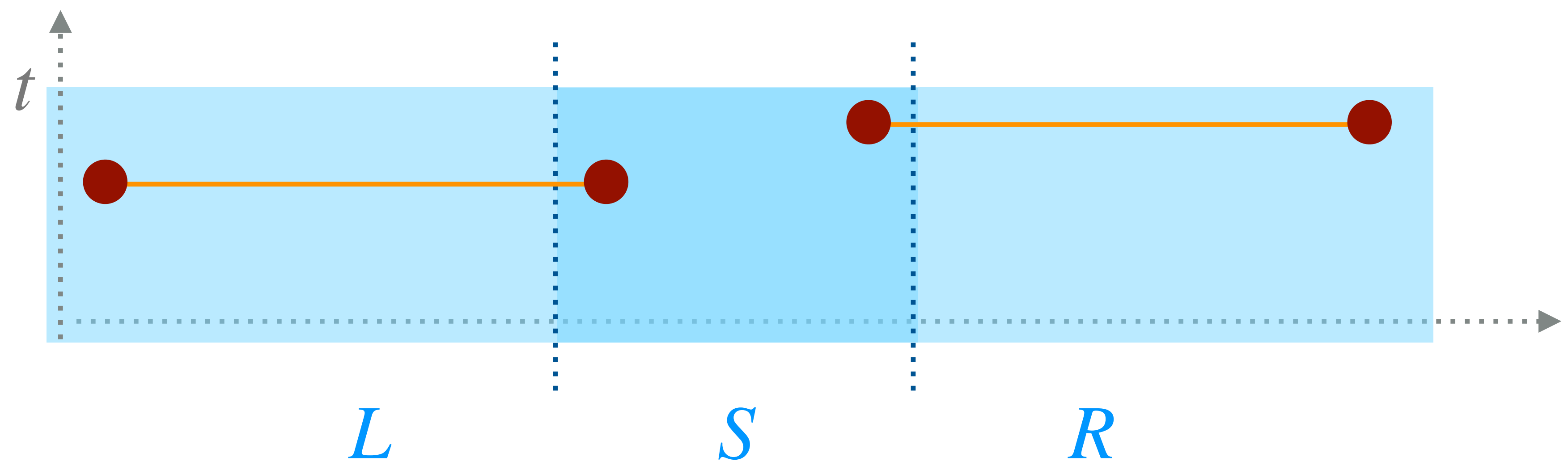


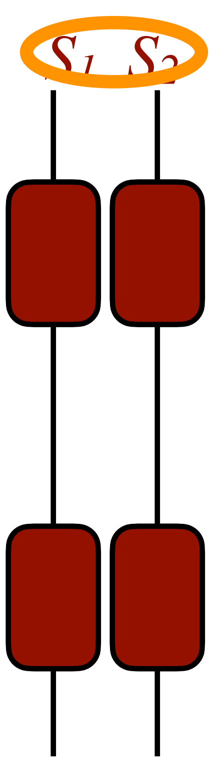


local observables on any S block

$|\Psi_{LSR}\rangle\langle\Psi_{LSR}|$

back to quasiparticle intuition

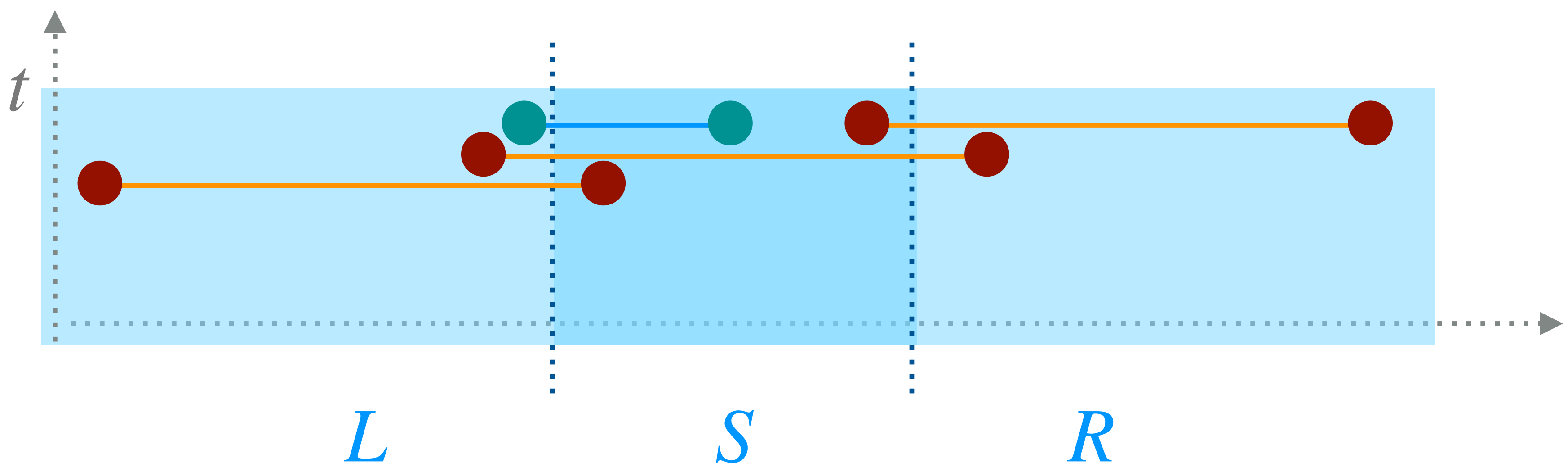




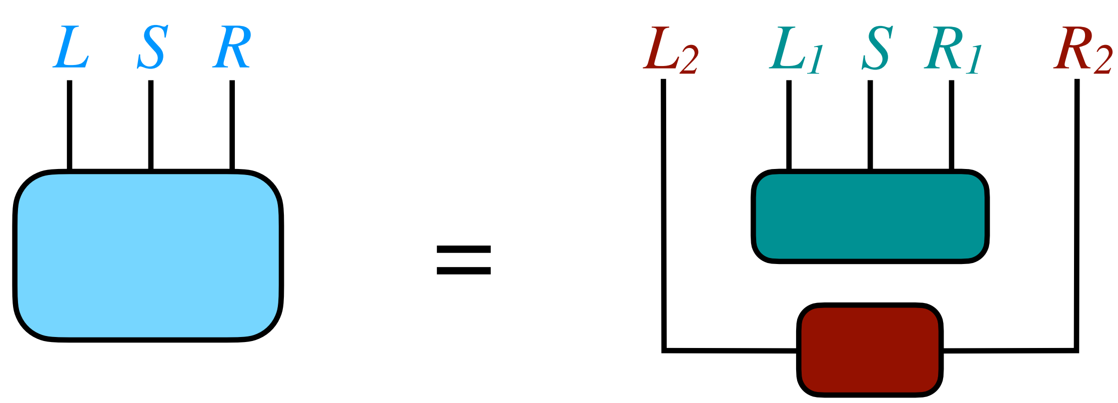
local observables on any S block

long-range entanglement contributes as statistical mixture

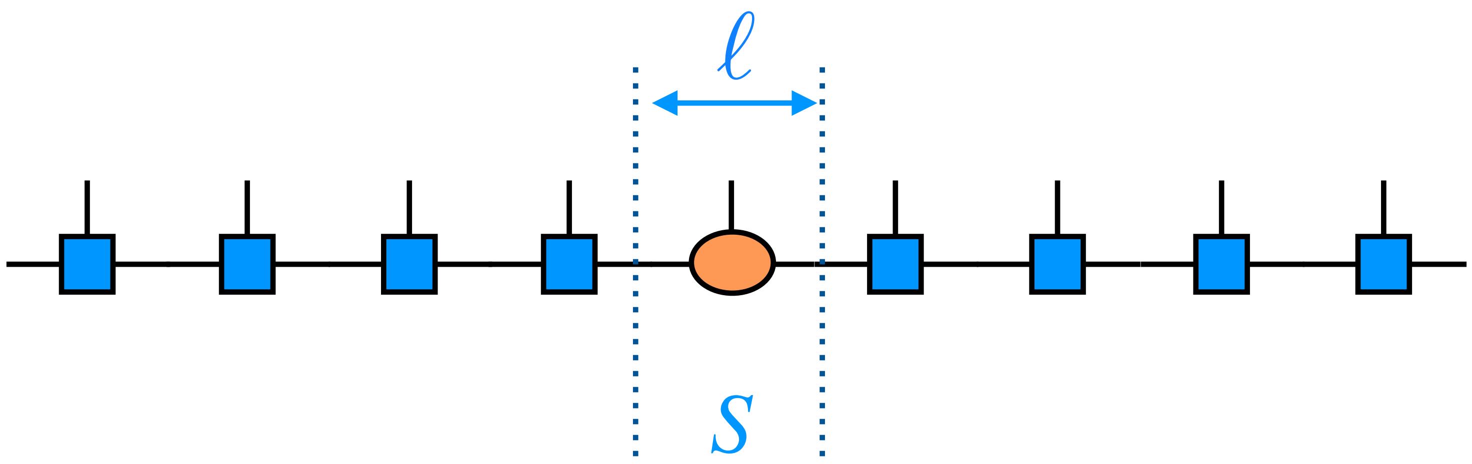
back to quasiparticle intuition

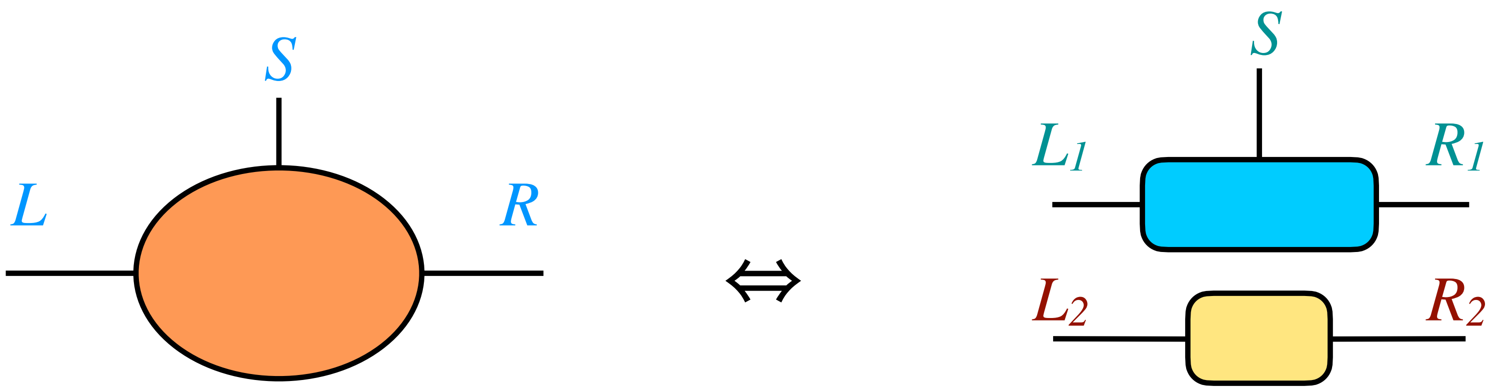


$$|\Psi_{LSR}\rangle = |\phi_{L_1SR_1}\rangle \otimes |\phi_{L_2R_2}\rangle$$



TN picture





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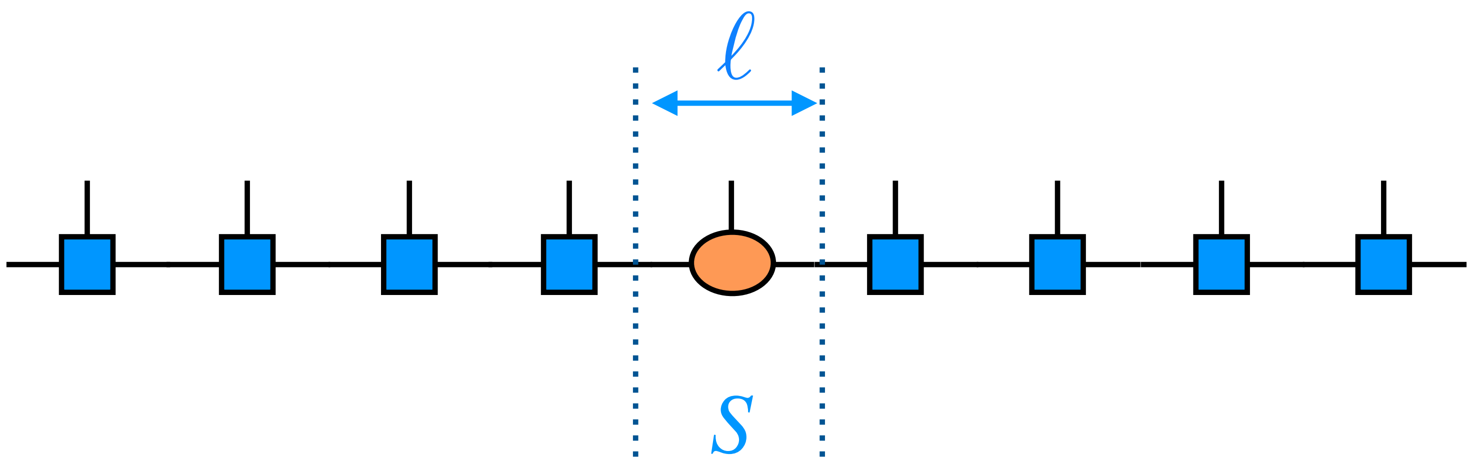
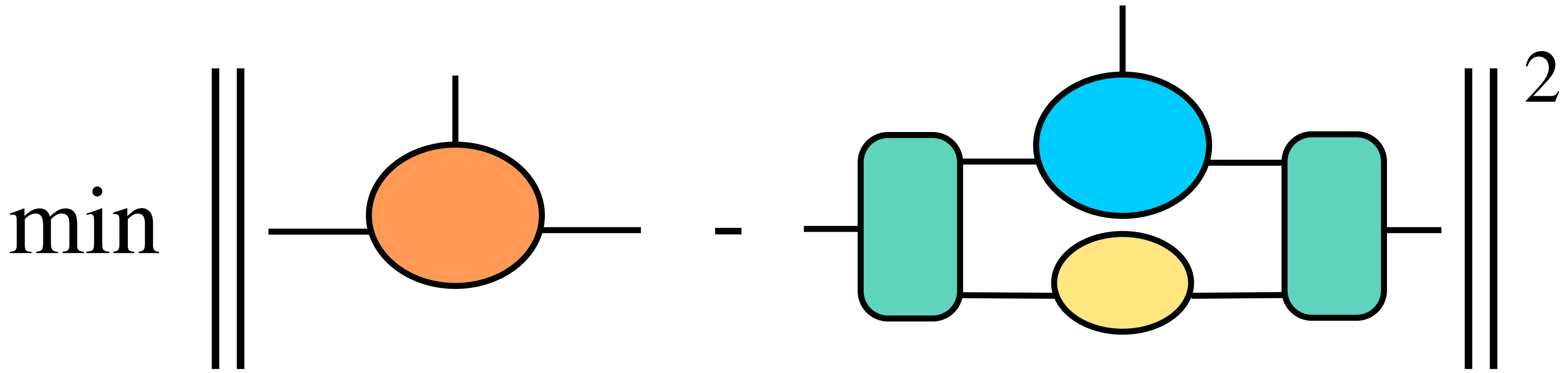
TN picture





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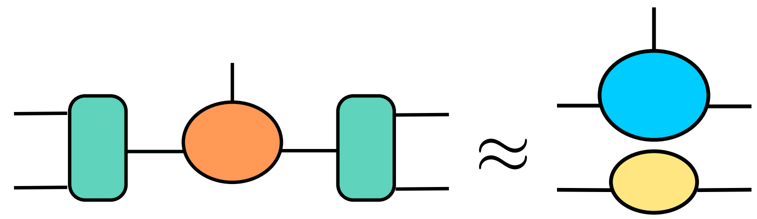
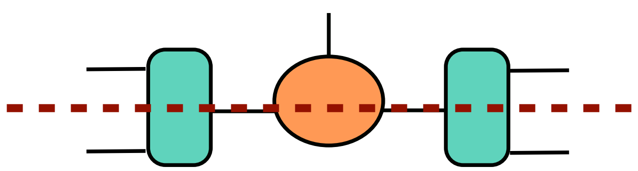
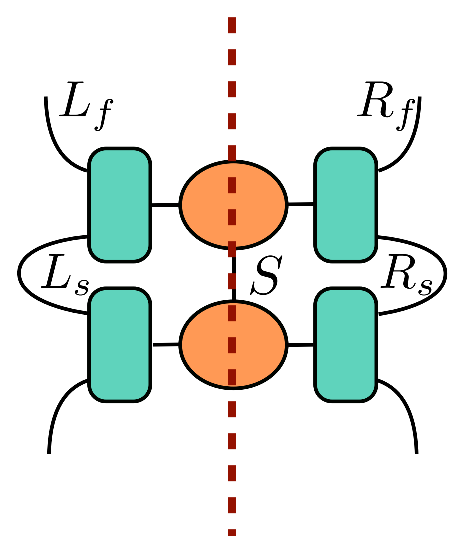
long-range entanglement in TN



$S_{\mathrm{fast-slow}}$ 

$\rho_{\mathrm{fast}} =$ 

$E_N(\rho_{\mathrm{fast}})$

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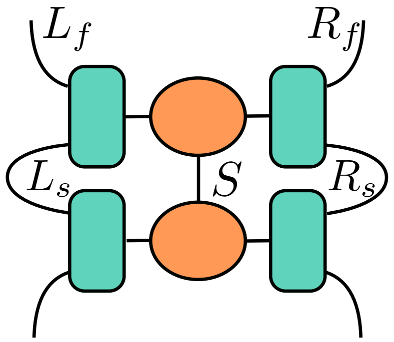
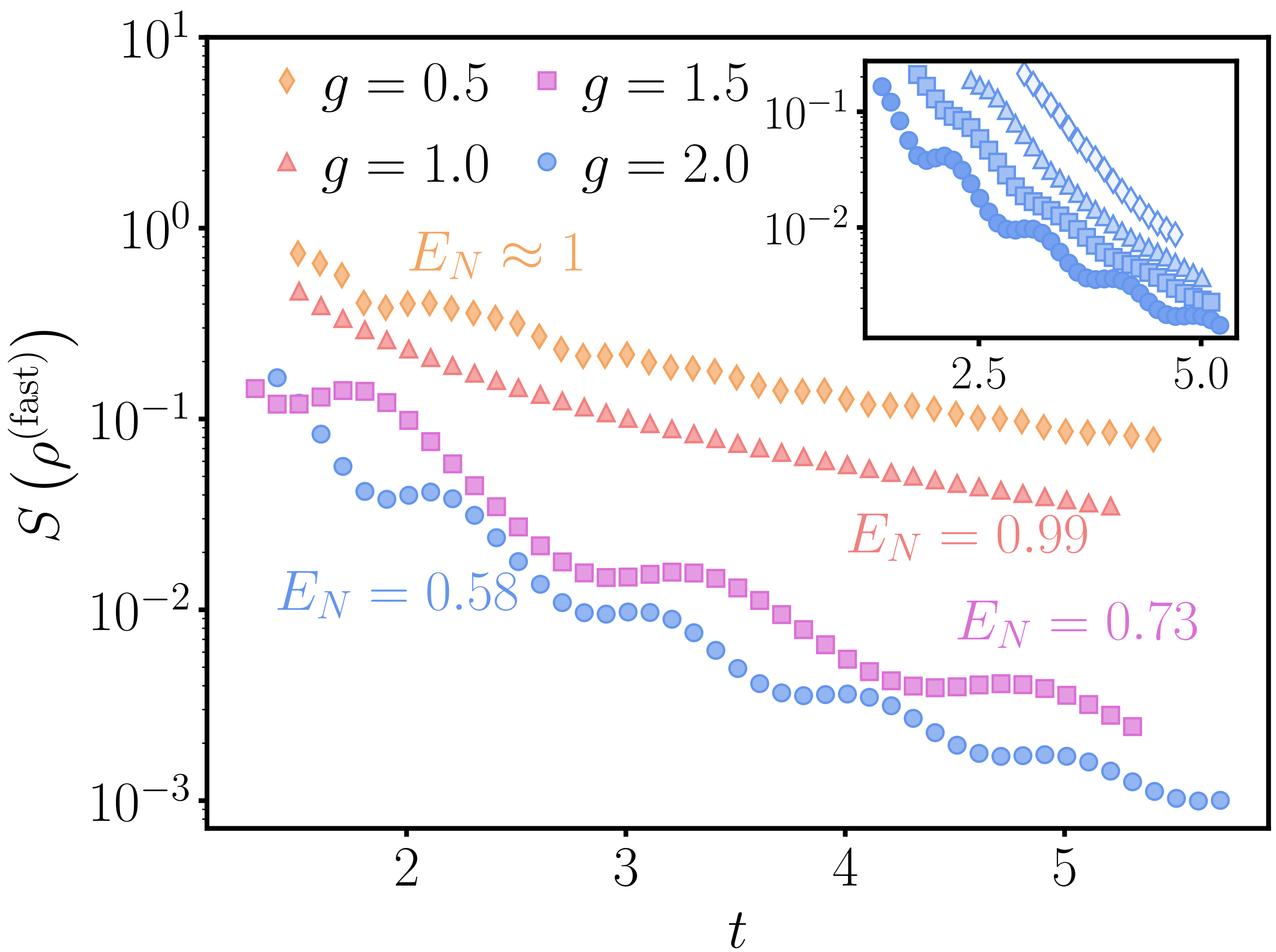
long-range entanglement in TN

$$H_{\text{Ising}} = -J\left(\sum_{i=1}^{N-1}\sigma_z^{[i]}\sigma_z^{[i+1]} + g\sum_{i}^{N}\sigma_x^{[i]}\right)$$

$\rho_{\text{fast}} =$ 



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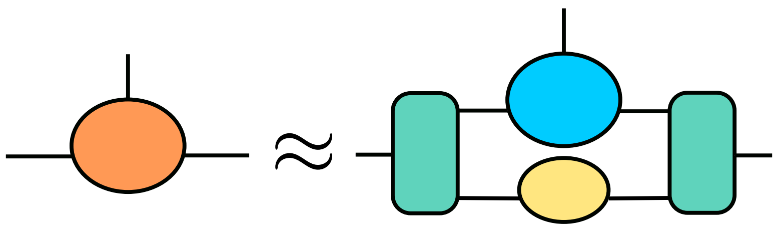
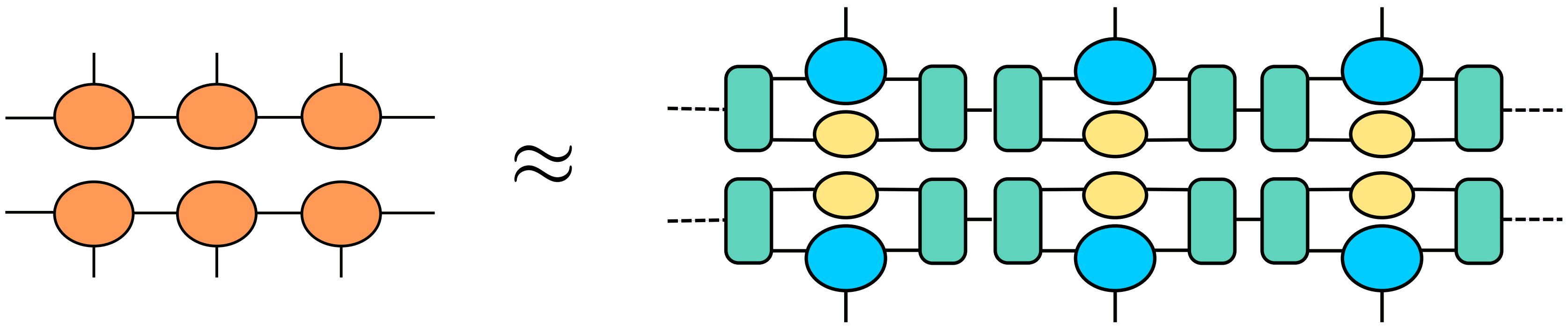
a TN algorithm

effective TN description



contribute as mixture to local observables in neighbouring blocks



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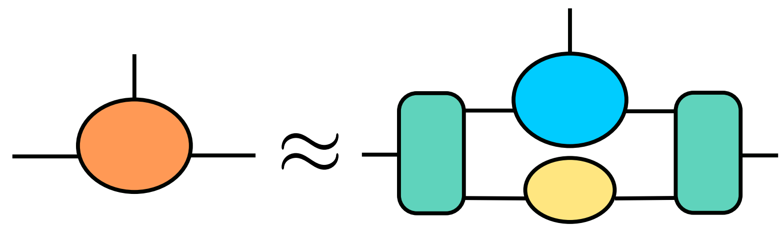
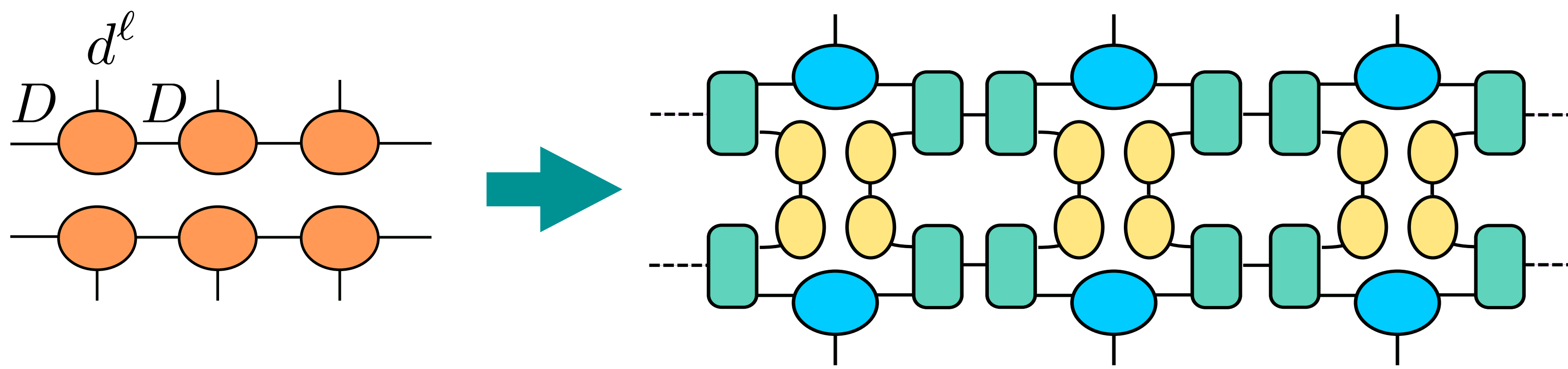
effective TN description



contribute as mixture to local
observables in neighbouring blocks



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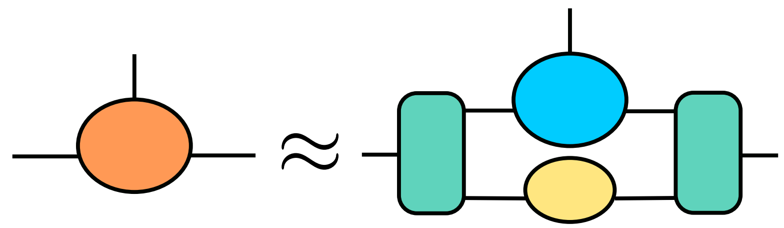
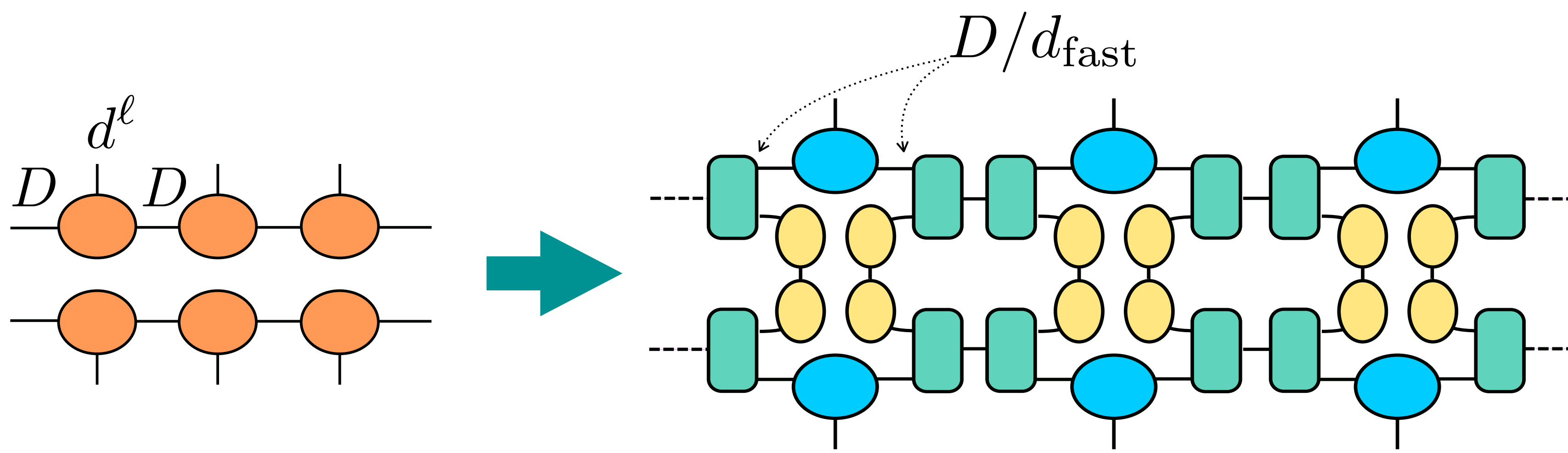
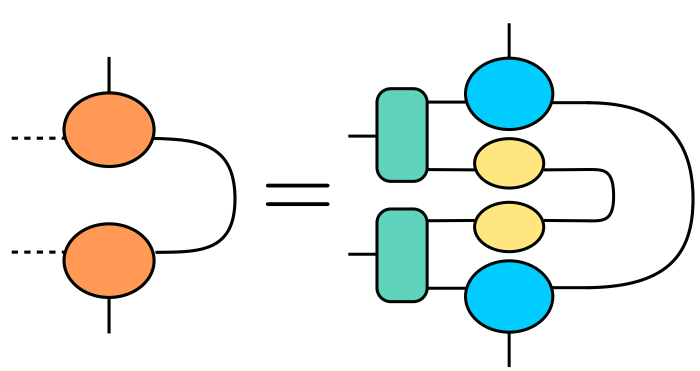
effective TN description

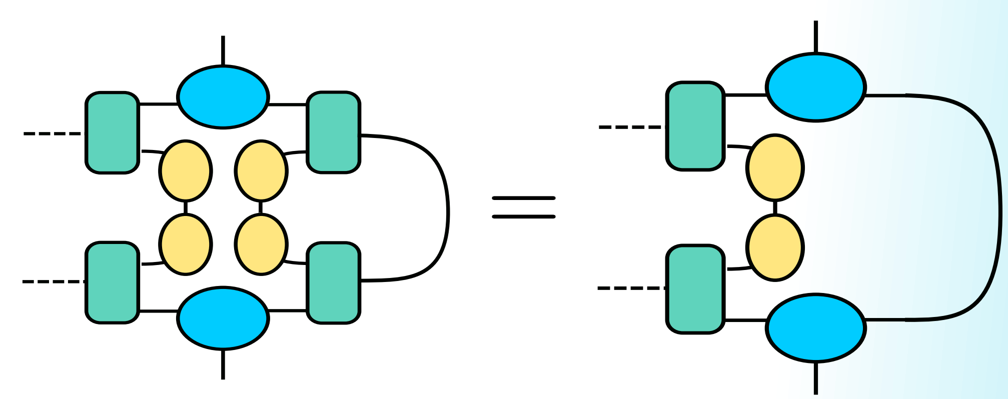


contribute as mixture to local observables in neighbouring blocks



preserves reduced density matrices





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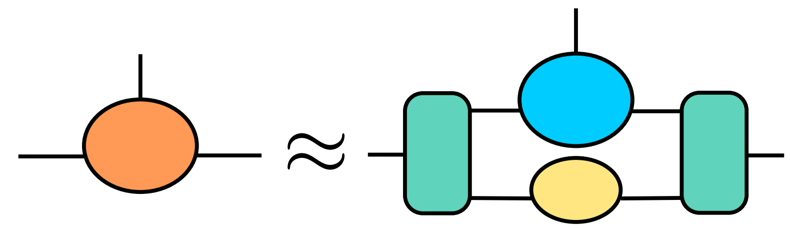
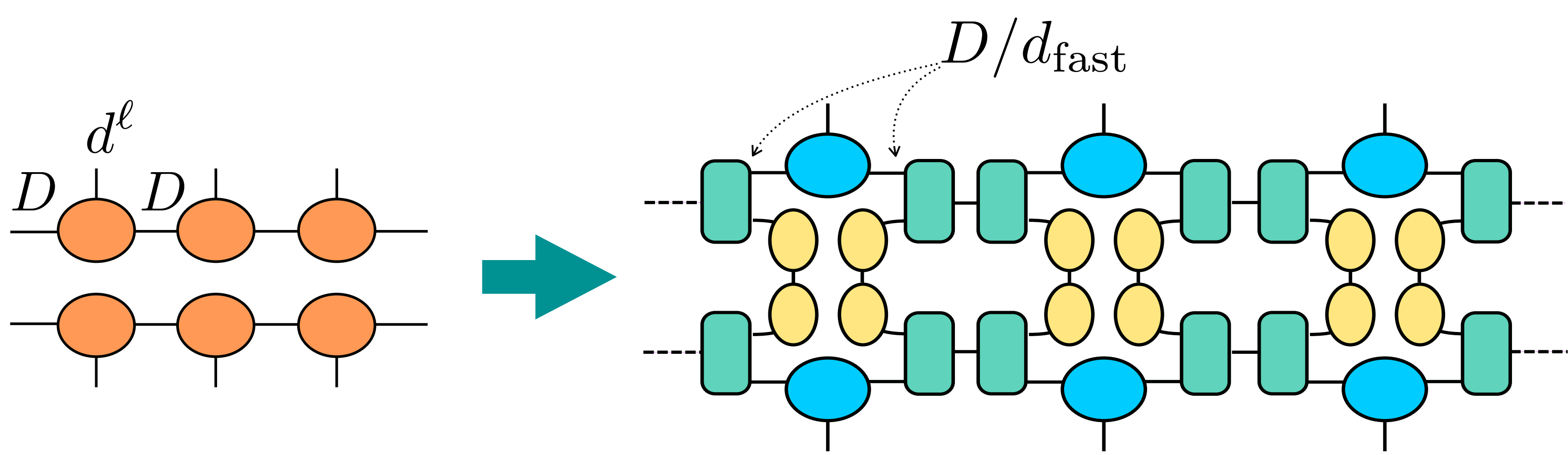
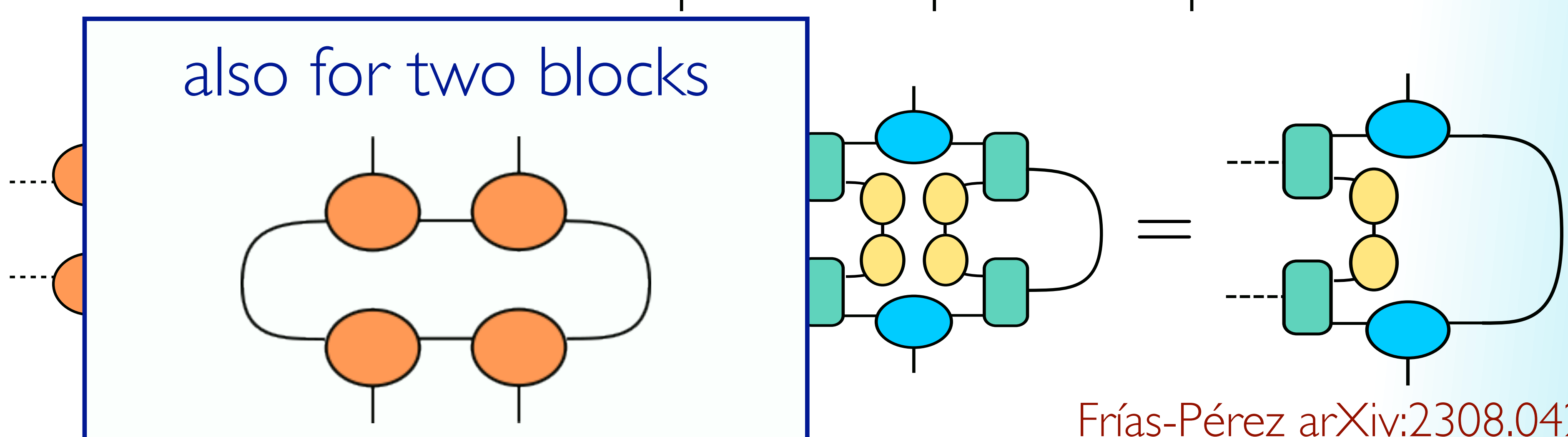
effective TN description



contribute as mixture to local observables in neighbouring blocks



preserves reduced density matrices



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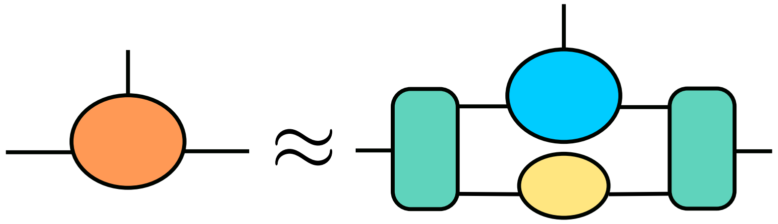
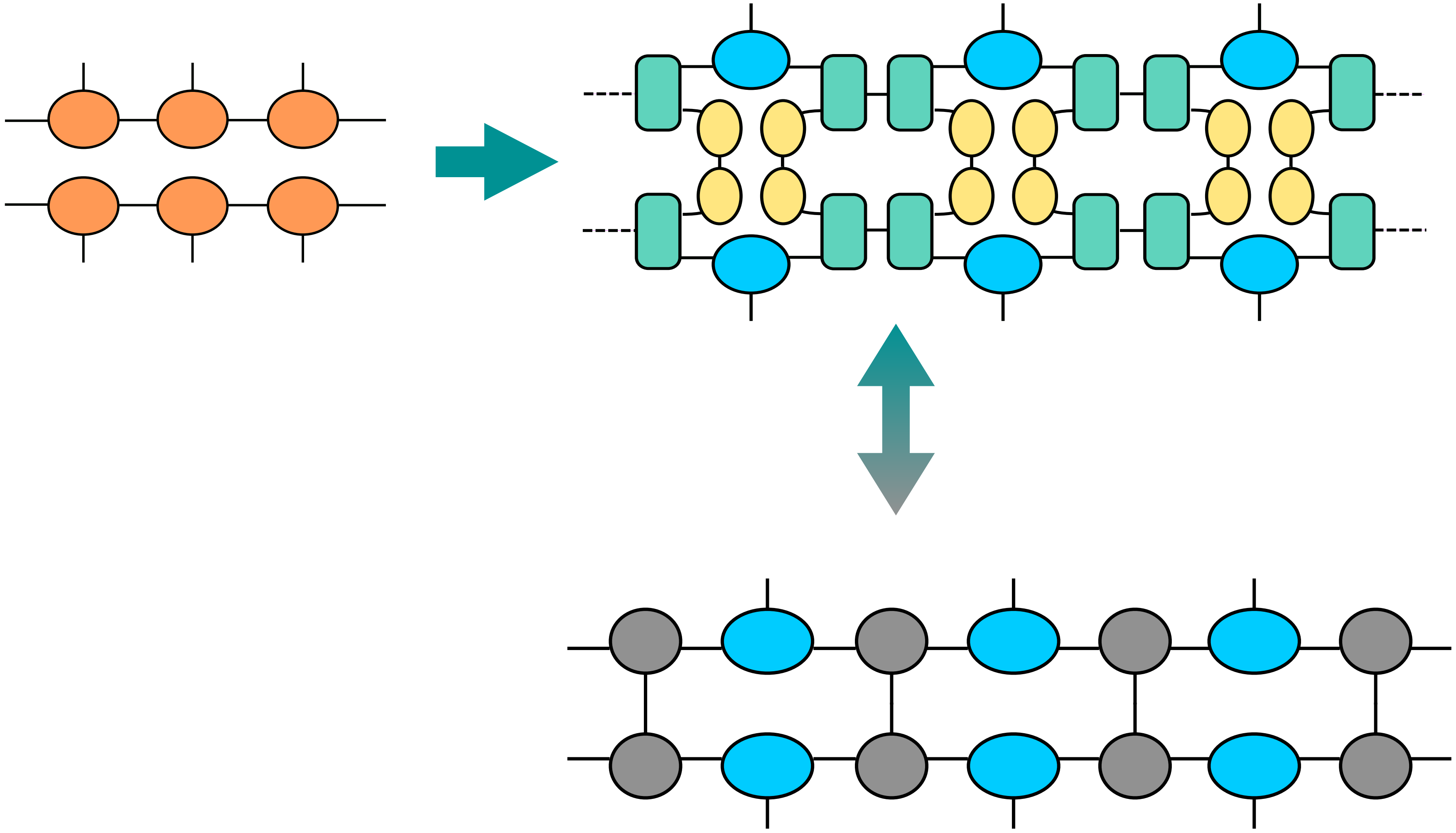
TN algorithm

1. 

2. 

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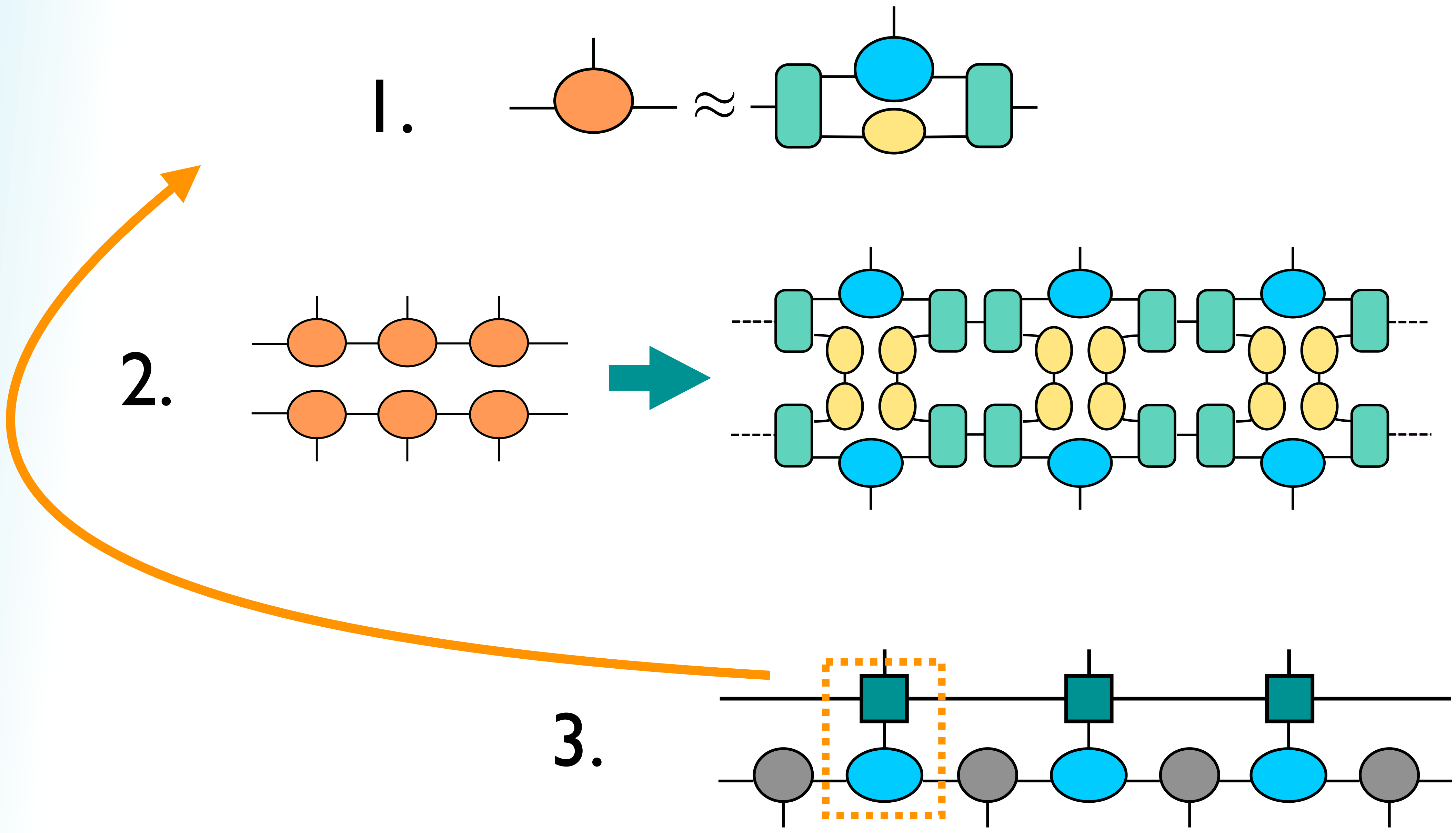
TN algorithm



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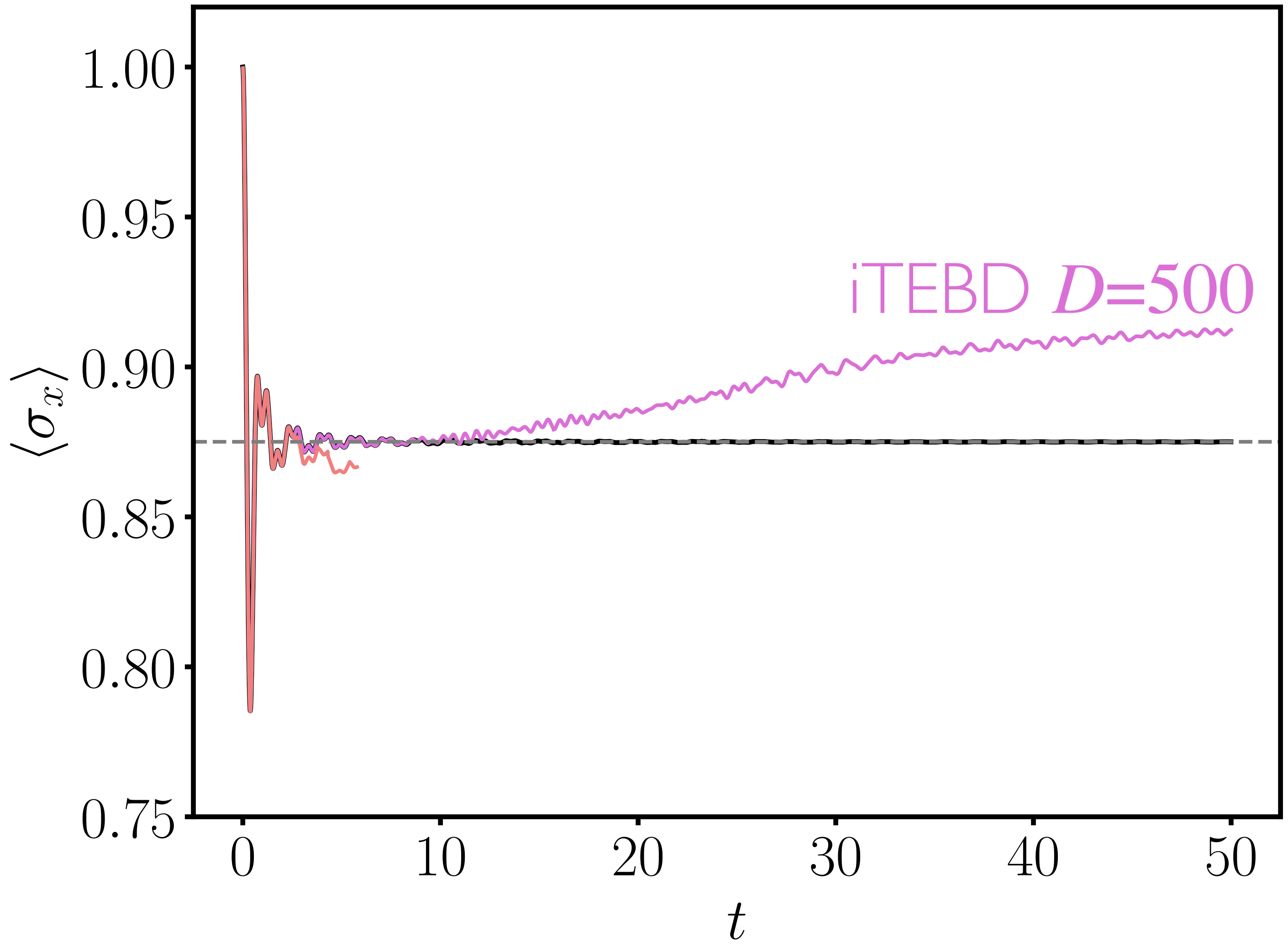
TN algorithm

$$H_{\text{Ising}} = -J\left(\sum_{i=1}^{N-1}\sigma_z^{[i]}\sigma_z^{[i+1]} + g\sum_{i}^{N}\sigma_x^{[i]}\right)$$

J=1

g=2



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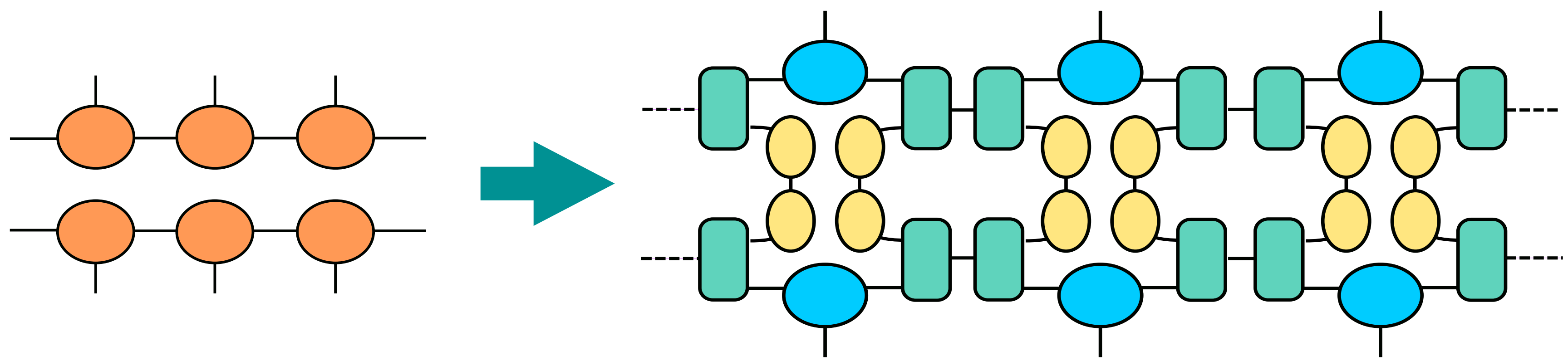
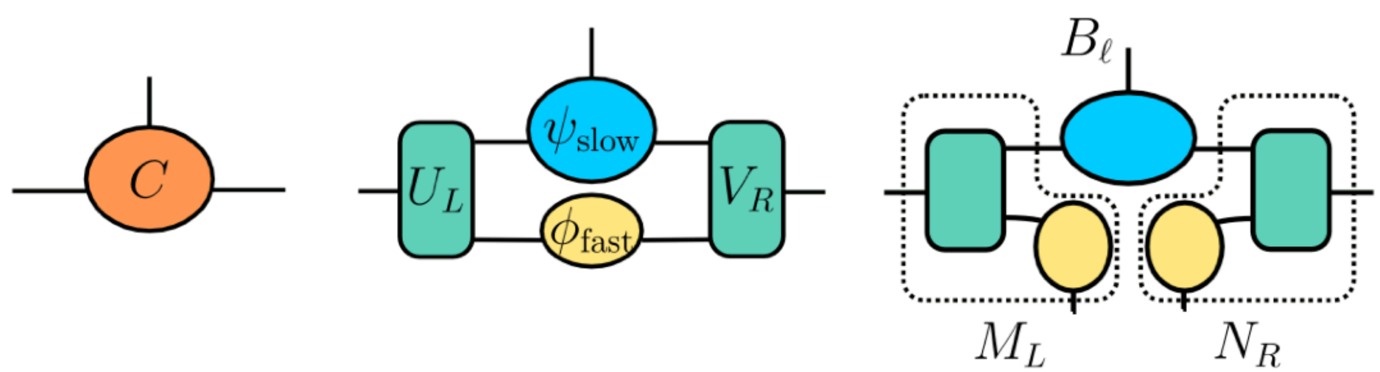
improved heuristic TN algorithm





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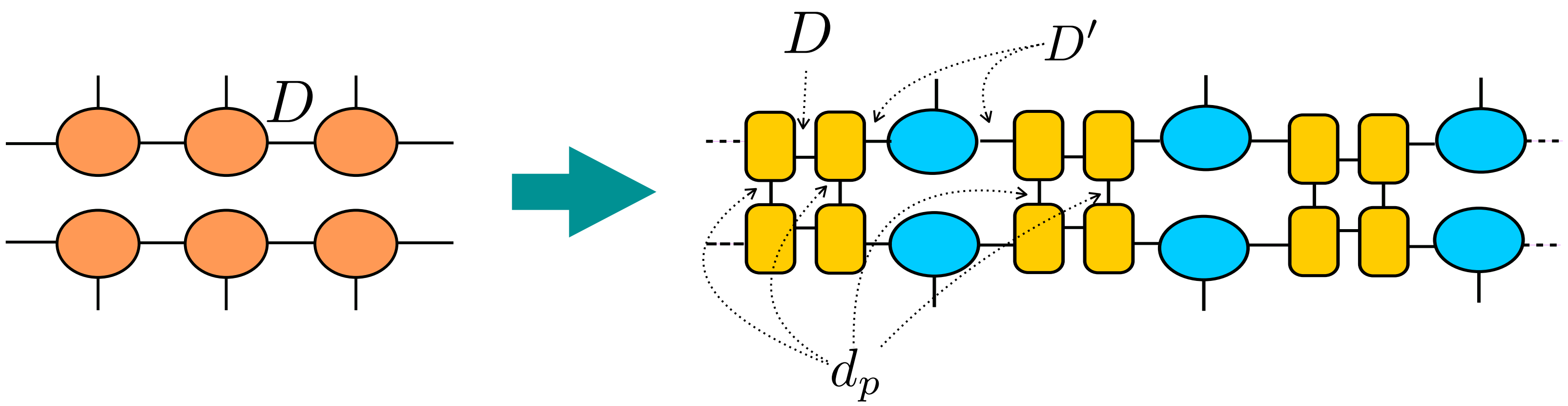
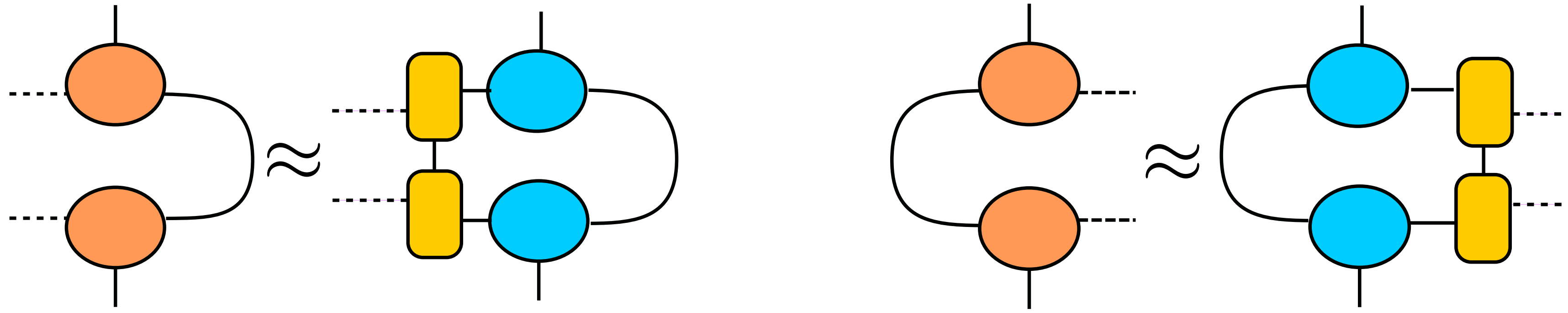
improved heuristic TN algorithm



such that condition on rdms holds



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TN algorithm

$$H_{\text{Ising}} = -J\left(\sum_{i=1}^{N-1}\sigma_z^{[i]}\sigma_z^{[i+1]} + g\sum_{i}^{N}\sigma_x^{[i]}\right)$$

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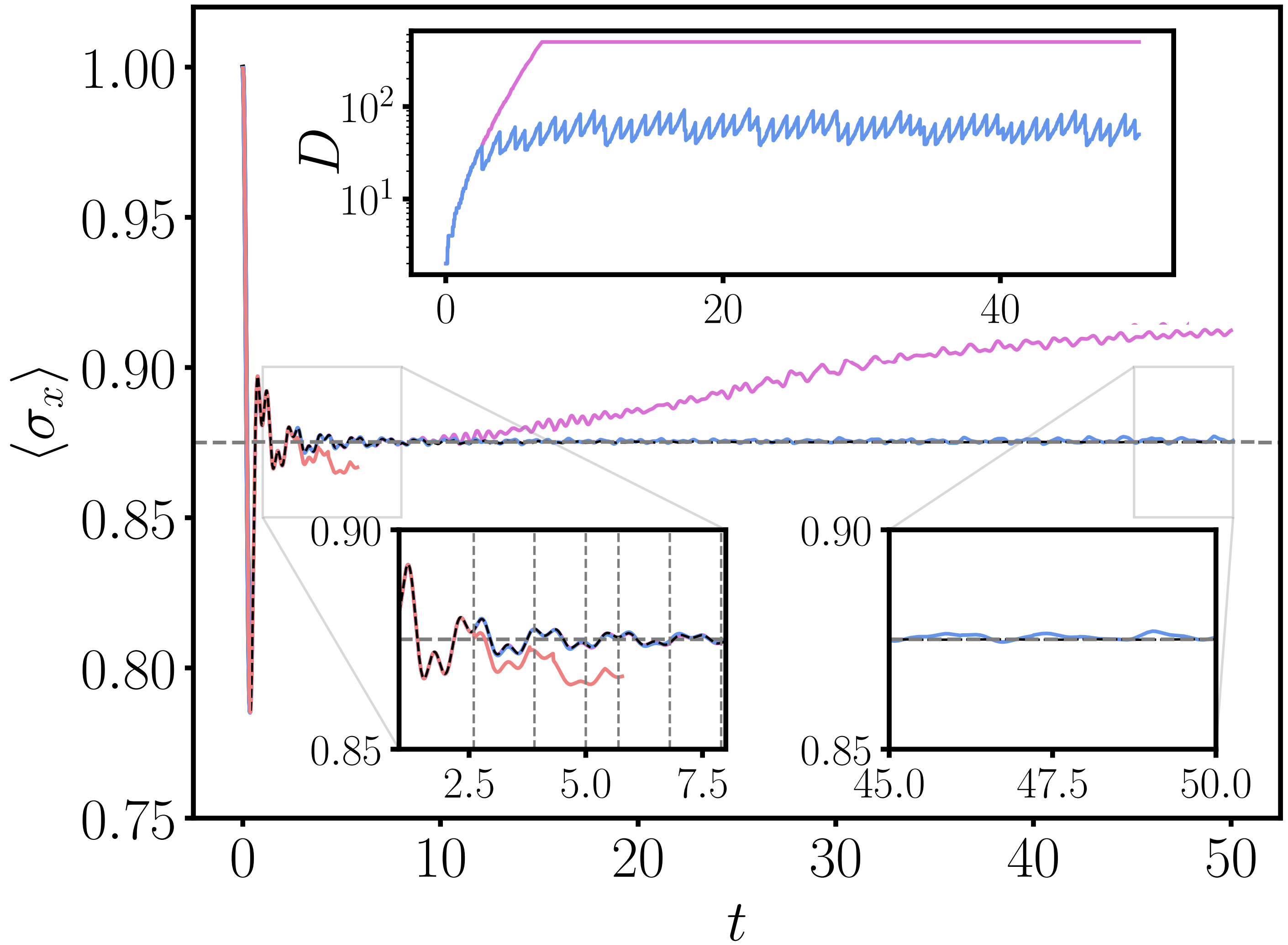

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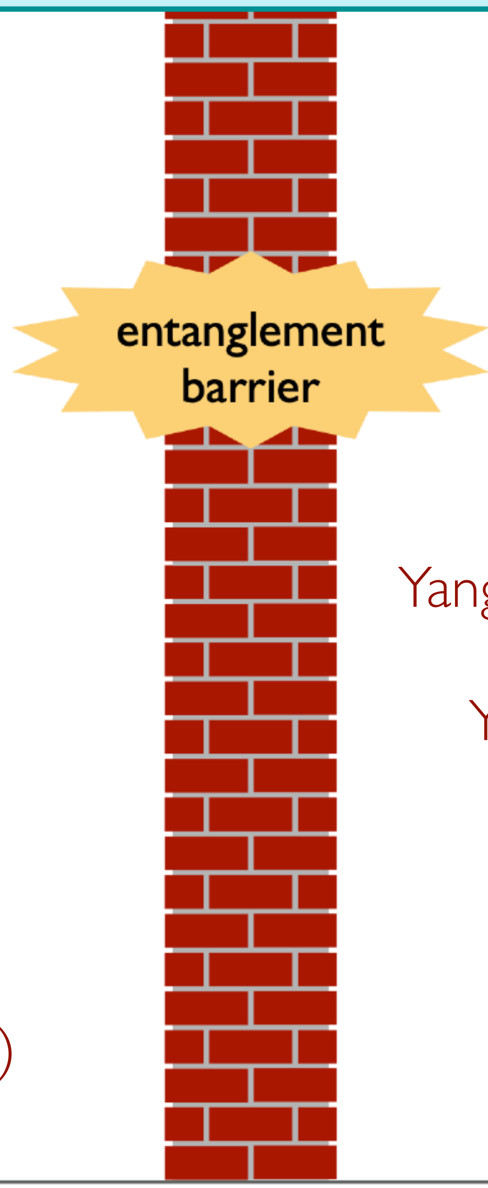
To conclude

energy filters & TNS can provide other (classical and quantum) tools to get dynamical properties



changing entanglement perspective:
transforming long-range entanglement into mixture

Frías-Pérez, Tagliacozzo, MCB, arXiv:2308.04291

light-cone TN contraction for local observables

Frías-Pérez, MCB, PRB 106, 115117 (2022)

spectral properties of a QMB Hamiltonian

Yang, Iblisdir, Cirac, MCB, PRL124, 100602 (2020)
Lu, PRX Quantum 2, 020321 (2021)
Yang, Cirac, MCB, PRB 106, 024307 (2022)